

Upper and lower bounds, **the biggest and smallest it could be**

Learn how to find upper and lower bounds using the rounding bucket method. Worked examples, exam-style questions with answers.

1 Half a step each way

2 Biggest $\times$ biggest makes the biggest

3 Big minus small, big $\div$ small

4 Chopped, not rounded

5 Practice + answers

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EXAM-STYLE
QUESTIONS

A plank is 2.4 m long to the nearest 10 cm. Could it actually be 2.46 m?

Upper and lower bounds feel hard because rounding hides real values. The rounding bucket holds all possible real lengths. You use the bucket edges to find the smallest and largest possible values.

The picture to keep in your head: The rounding bucket

Rounding drops every real length into a bucket. Everything from 2.35 m up to (but not including) 2.45 m lands in the '2.4 m' bucket. The edges of the bucket are the bounds: half the rounding step below and above. When you combine measurements, you pick from the edges of the buckets to make the answer as big, or as small, as it could be.

The method, every time

1. Find half the unit size. Add and subtract it from the rounded number.
2. Use the upper bound for multiplication and division. Use the lower bound for division.
3. Use the upper bound for addition and subtraction. Use the lower bound for subtraction.
4. Keep full values until the end. Only round at the end to find the final answer.
5. Check your answer by using the bucket edges for the biggest and smallest possible results.

Words you'll see: **bounds** = Smallest and largest possible values from a rounded measurement. · **bucket** = A range of real values that round to a given number. · **unit** = The size of the rounding step, like 10 cm or 1 cm. · **truncation** = Cutting off the end of a number without rounding. · **interval** = The range of values between the lower and upper bounds.

Half a step each way

The rounding bucket holds all lengths that round to a given value. To find the error interval, you halve the rounding unit and add or subtract it. This gives the smallest and largest possible values. The error interval is written as lower $\leq x <$ upper.

WORKED EXAMPLE A length is 8.6 cm to 1 decimal place. Write its error interval.

8.6 cm

Original measurement rounded to 1 decimal place

0.1 cm

Rounding unit is 1 decimal place

0.05 cm

Halve the rounding unit

$$8.6 - 0.05 = 8.55$$

Subtract half the unit for lower bound

$$8.6 + 0.05 = 8.65$$

Add half the unit for upper bound

$$8.55 \leq x < 8.65$$

Write the error interval with bounds

$$8.55 \leq x < 8.65$$

answer

Watch out

- **Using the whole rounding unit instead of half** Always halve the rounding unit before adding or subtracting
- **Writing $\leq$ at the upper end** Use $<$ at the upper end of the error interval

Biggest × biggest makes the biggest

The rounding bucket holds all real lengths that round to a given value. To find the biggest area, use the biggest lengths from each bucket. For the largest area, pick the upper bounds of both measurements. This means using the biggest possible values for each measurement before multiplying.

WORKED EXAMPLE A rectangle is 12 cm by 5 cm, both to the nearest cm. Find the upper bound of its area.

length = 12 cm (nearest cm)

Given measurement with rounding unit

upper bound = 12.5 cm

Half the rounding step above the value

width = 5 cm (nearest cm)

Given measurement with rounding unit

upper bound = 5.5 cm

Half the rounding step above the value

area = 12.5×5.5

Multiply the upper bounds for maximum area

area = 68.75 cm²

Calculate the product of the upper bounds

68.75

answer

Watch out

- **Rounding the bounds before multiplying** Use the exact upper bounds before multiplying
- **Using a lower and an upper bound together** Always use both upper or both lower bounds together

Big minus small, big $\div$ small

The rounding bucket helps us find the biggest and smallest possible values. For big minus small or big $\div$ small, use the upper bound of the first and lower bound of the second. This gives the biggest difference or speed. To find the smallest, swap the bounds.

WORKED EXAMPLE A car travels 150 km (nearest 10 km) in 2.5 hours (nearest 0.1 hour). Find the upper bound for its average speed, to 2 decimal places.

upper distance = 155 km

150 to the nearest 10: add 5

lower time = 2.45 h

2.5 to the nearest 0.1: take 0.05

$155 \div 2.45$

Biggest $\div$ smallest gives the biggest speed

63.265...

Keep the full value

63.27

Round only at the end

63.27

answer

Watch out

- **Dividing upper by upper** Use upper for first and lower for second to get biggest result
- **Rounding the final answer too early** Only round at the end to keep accuracy

Chopped, not rounded

Truncating chops digits off without rounding: 5.79 truncated to 1 decimal place is still 5.7. So everything from 5.7 up to (but not including) 5.8 truncates to 5.7. The bucket only goes upwards from the value, a whole step, not half a step each way.

WORKED EXAMPLE $x = 5.7$ truncated to 1 decimal place. Write the error interval for x .

$$5.7 \leq x$$

5.7 itself truncates to 5.7

$$x < 5.8$$

Anything from 5.8 up truncates to 5.8

$$5.7 \leq x < 5.8$$

A whole step upwards

$$5.7 \leq x < 5.8$$

answer

Watch out

- **Treating truncation like rounding** Truncation cuts off digits, not rounds them
- **Giving more accuracy than the bounds support** Use the bounds as they are, don't add extra digits

Exam-style questions

Show your working: most marks are for the method. F = Foundation / IGCSE Core, H = Higher / IGCSE Extended. Answers at the back.

A · ERROR INTERVALS

- 1** A number is rounded to the nearest 10. The result is 30. Write down the error interval for the original number. F [2]

- 2** A number is rounded to the nearest whole number. The result is 7. Write down the error interval for the original number. F [2]

- 3** A number is rounded to 1 decimal place. The result is 4.7. Write down the error interval for the original number. F [2]

- 4** A number is rounded to 2 significant figures. The result is 3.6. Write down the error interval for the original number. F [2]

- 5** A number is rounded to the nearest whole number. The result is 12. Write down the error interval for the original number. F [2]

B · SUMS AND PRODUCTS

- 6** A rectangle has length 8.2 cm and width 4.1 cm, both rounded to the nearest 0.1 cm. Calculate the upper and lower bounds of the perimeter. F/H [3]

- 7** A rectangle has length 8.2 cm and width 4.1 cm, both rounded to the nearest 0.1 cm. Calculate the upper and lower bounds of the area. F/H [3]

- 8** 20 identical bottles each have a mass of 0.5 kg, rounded to the nearest 0.1 kg. Find the upper and lower bounds of the total mass. F/H [3]

- 9** A square has side length 6.3 cm, rounded to the nearest 0.1 cm. Find the upper and lower bounds of the area. F/H [3]

C · DIFFERENCES AND DIVISION

- 10** A car travels 120 km (to the nearest km) in 2 hours (to the nearest 0.1 hour). Find the upper bound of its average speed, to 2 decimal places. H [3]

- 11** A block has mass 2.5 kg (to 1 decimal place) and volume 0.35 m³ (to 2 decimal places). Find the lower bound of its density, to 2 decimal places. H [3]

- 12** Two rods are 1.4 m and 2.6 m long, both to the nearest 0.1 m. Find the upper bound of the difference in their lengths. H [3]

- 13** Two rods are 1.2 m and 2.3 m long, both to the nearest 0.1 m. Find the lower bound of the difference in their lengths. H [3]
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- 14** A rectangle has area 50 cm^2 (to the nearest cm^2) and width 4 cm (to the nearest cm). Find the upper bound of its length, to 2 decimal places. H [3]
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D • TRUNCATION AND ACCURACY

- 15** A number is truncated to the nearest whole number. The result is 7. Write down the error interval for the original number. H [3]
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- 16** A number is truncated to 1 decimal place. The result is 4.7. Write down the error interval for the original number. H [3]
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- 17** $x = 3.4$ and $y = 1.2$, both to 1 decimal place. Work out $x \div y$ to a suitable degree of accuracy. Give a reason. H [3]
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- 18** A number is truncated to the nearest 10. The result is 30. Write down the error interval for the original number. H [3]
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E • REAL PROBLEMS

- 19** Will a 1.85 m sofa (nearest 5 cm) definitely fit in a 1.9 m gap (nearest 10 cm)? Explain using bounds. H [4]
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- 20** How many 250 ml cups (nearest 10 ml) can definitely be filled from 5 litres (nearest 0.1 litre)? H [4]
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- 21** A 2.5 L container (nearest 0.1 L) is used to fill 1200 mL jugs (nearest 10 mL). How many jugs can definitely be filled? H [4]
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Marks are a guide to how much working an exam would expect. Questions written by Bloom Education in the style of GCSE (AQA, Edexcel, OCR) and Cambridge IGCSE papers.

Check your work

1 $25 \leq x < 35$ The rounding step is 10. The lower bound is $30 - 5 = 25$. The upper bound is $30 + 5 = 35$. So $25 \leq x < 35$.

2 $6.5 \leq x < 7.5$ The rounding step is 1. The lower bound is $7 - 0.5 = 6.5$. The upper bound is $7 + 0.5 = 7.5$. So $6.5 \leq x < 7.5$.

3 $4.65 \leq x < 4.75$ The rounding step is 0.1. The lower bound is $4.7 - 0.05 = 4.65$. The upper bound is $4.7 + 0.05 = 4.75$. So $4.65 \leq x < 4.75$.

4 $3.55 \leq x < 3.65$ The rounding step is 0.1. The lower bound is $3.6 - 0.05 = 3.55$. The upper bound is $3.6 + 0.05 = 3.65$. So $3.55 \leq x < 3.65$.

5 $11.5 \leq x < 12.5$ The rounding step is 1. The lower bound is $12 - 0.5 = 11.5$. The upper bound is $12 + 0.5 = 12.5$. So $11.5 \leq x < 12.5$.

6 Upper bound of the perimeter: 24.8 cm. Lower bound of the perimeter: 24.4 cm. Perimeter = $2 \times (\text{length} + \text{width})$. Upper bound: $2 \times (8.25 + 4.15) = 2 \times 12.4 = 24.8$ cm. Lower bound: $2 \times (8.15 + 4.05) = 2 \times 12.2 = 24.4$ cm.

7 Upper 34.2375 cm², lower 33.0075 cm² Upper: $8.25 \times 4.15 = 34.2375$. Lower: $8.15 \times 4.05 = 33.0075$.

8 Upper bound: 11.0 kg, Lower bound: 9.0 kg Total mass = $20 \times \text{mass}$. Upper bound: $20 \times 0.55 = 11.0$ kg. Lower bound: $20 \times 0.45 = 9.0$ kg.

9 Upper bound: 40.3225 cm², Lower bound: 39.0625 cm² Area = side $\times$ side. Upper bound: $6.35 \times 6.35 = 40.3225$ cm². Lower bound: $6.25 \times 6.25 = 39.0625$ cm².

10 61.79 km/h Upper distance $\div$ lower time = $120.5 \div 1.95 = 61.79\dots$

11 6.90 kg/m³ Lower mass $\div$ upper volume = $2.45 \div 0.355 = 6.901\dots$

12 1.3 m Upper of the longer minus lower of the shorter: $2.65 - 1.35$.

13 1.0 m Lower of the longer minus upper of the shorter: $2.25 - 1.25$.

14 14.43 cm Length = area $\div$ width: upper area $\div$ lower width = $50.5 \div 3.5 = 14.428\dots$

15 $7 \leq x < 8$ Truncating chops off the decimals, so everything from 7 up to (not including) 8 becomes 7.

16 $4.7 \leq x < 4.8$ Truncating to 1 decimal place means cutting off after the first decimal. The number is between 4.7 and 4.799... The lower bound is 4.7, upper bound is 4.8. Error interval is $4.7 \leq x < 4.8$.

17 3, to 1 significant figure Upper $3.45 \div 1.15 = 3.0$, lower $3.35 \div 1.25 = 2.68$; both round to 3 at 1 s.f., but not at 2 s.f.

18 $30 \leq x < 40$ Truncating to the nearest 10 chops off the units: everything from 30 up to (not including) 40 becomes 30.

19 No, it may not fit Longest sofa 1.875 m; narrowest gap 1.85 m. $1.875 > 1.85$, so it might not fit.

20 19 cups Least water 4950 ml $\div$ biggest cup 255 ml = 19.4..., so 19 cups for certain.

21 2 jugs Least water 2450 ml $\div$ biggest jug 1205 ml = 2.03..., so 2 jugs for certain.

Exam checklist

- Half the unit each way to find the bucket edges.
- Lower $\leq x <$ upper to show the range of possible values.
- Sum and product: upper with upper for the biggest result.
- Difference and division: upper with lower for the smallest result.
- Keep full values until the end to avoid errors.
- Truncation only goes up, not down, so watch out for that.
- Use the bucket edges to make the answer as big or as small as it could be.

Got stuck on a question?

- Find the section it belongs to and go through its worked example again, line by line.
- Put your answer back into the question. If it doesn't work, look for a sign or a step you skipped.
- Still stuck? Message us on Instagram (@bloom_education_international). We read every message.

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