

Surds, **made simple**

Learn how to simplify surds with the square-root bouncer analogy. Worked examples, exam-style questions, and a checklist to help you simplify and rationalise surds.

1 Let the square numbers out

2 Roots can share a room

3 Only like surds add up

4 Clear the root from the bottom

5 Practice + answers

23
EXAM-STYLE
QUESTIONS

Why does your calculator turn $\sqrt{12}$ into $2\sqrt{3}$, and why is that the same number?

Surds feel hard because they look messy. Think of the square-root bouncer who only lets square numbers out. This helps simplify roots into exact forms, not long decimals.

The picture to keep in your head: The square-root bouncer

Think of the square-root sign as a club with a bouncer. Only square numbers (4, 9, 16, 25...) are allowed out, and when one leaves it becomes its root: a 4 walks out as a 2. So $\sqrt{12} = \sqrt{(4 \times 3)}$: the 4 leaves as 2, the 3 can't leave, so it's $2\sqrt{3}$. A surd is just a root that can't fully leave the club, so we keep it exact instead of writing a long decimal.

The method, every time

1. Let the square numbers out first. Find the biggest square that divides the number under the root.
2. Share the room with like surds. Only roots with the same number can be added or subtracted.
3. Multiply roots together. When you multiply two roots, you can combine them under one square root.
4. Clear the root from the bottom. Multiply top and bottom by the same root to make the denominator nice.
5. Write the answer in its simplest form. Make sure no square numbers are left inside the root.

Words you'll see: **surd** = A root that can't fully leave the club, so we keep it exact. · **simplify** = Make the surd look neater by letting square numbers out first. · **rationalise** = Remove the root from the bottom of a fraction by multiplying top and bottom.

Let the square numbers out

The square-root bouncer only lets square numbers out. So, split the number under the root into the biggest square number times what's left. Then the square number leaves as its root. This way, you simplify the surd properly.

WORKED EXAMPLE Simplify $\sqrt{72}$

$$\sqrt{72}$$

Start with the question

$$\sqrt{(36 \times 2)}$$

Split 72 into the biggest square number 36 and 2

$$\sqrt{36} \times \sqrt{2}$$

Split the root into two parts

$$6 \times \sqrt{2}$$

36 leaves as 6, 2 can't leave

$$6\sqrt{2}$$

Write the answer

$$6\sqrt{2}$$

answer

Watch out

- **Using a square number that isn't the biggest one, e.g. $\sqrt{72} = 2\sqrt{18}$ and stopping**
Always find the biggest square number first
- **Writing $\sqrt{12} = 3\sqrt{4}$** Check if the square number is actually a factor of the whole number

Roots can share a room

The square-root bouncer lets two roots share a room. When you multiply or divide roots, they merge into one. If the same root is multiplied, it becomes a square and walks out. Multiply the numbers outside and the roots inside. This is how you simplify expressions with surds.

WORKED EXAMPLE Simplify $3\sqrt{2} \times 5\sqrt{6}$

$$3\sqrt{2} \times 5\sqrt{6}$$

$$3 \times 5$$

$$15$$

$$\sqrt{2} \times \sqrt{6}$$

$$\sqrt{(2 \times 6)}$$

$$\sqrt{12}$$

$$\sqrt{(4 \times 3)}$$

$$2\sqrt{3}$$

$$30\sqrt{3}$$

Start with the question

Multiply the numbers outside the roots

$$3 \times 5 = 15$$

Multiply the roots inside

Multiply the numbers under the roots

Combine the roots into one

Split 12 into a square and a non-square

The 4 walks out as 2, the 3 stays inside

answer

Watch out

- **Adding instead of multiplying the outside numbers** Multiply the numbers outside, not add them
- **Forgetting to simplify the root at the end** Always check if the root can be simplified

Only like surds add up

The square-root bouncer only lets like surds out together. Think of $2\sqrt{3} + 5\sqrt{3}$ as 2 apples + 5 apples = 7 apples. But $\sqrt{2} + \sqrt{3}$ is like apples and pears — they can't be added. Always simplify surds first to see if they're like terms.

WORKED EXAMPLE Simplify $\sqrt{50} + \sqrt{18}$

$$\sqrt{50} + \sqrt{18}$$

Start with the question

$$\sqrt{(25 \times 2)} + \sqrt{(9 \times 2)}$$

Split into square numbers and surds

$$\sqrt{25} \times \sqrt{2} + \sqrt{9} \times \sqrt{2}$$

Use the rule $\sqrt{(ab)} = \sqrt{(a)} \times \sqrt{(b)}$

$$5 \times \sqrt{2} + 3 \times \sqrt{2}$$

Simplify the square roots

$$5\sqrt{2} + 3\sqrt{2}$$

Write in simplest form

$$8\sqrt{2}$$

Add the coefficients of like surds

$$8\sqrt{2}$$

answer

Watch out

- **Writing $\sqrt{2} + \sqrt{3} = \sqrt{5}$** You can't add surds like numbers — they're different types.
- **Not simplifying each surd first** Always simplify surds before trying to add them.

Clear the root from the bottom

The square-root bouncer won't let a root stay at the bottom of a fraction. To tidy it up, you multiply top and bottom by the same root. That way, the root leaves the denominator and joins the numerator. This is called rationalising the denominator. You can only do this if the root is in the denominator.

WORKED EXAMPLE Rationalise the denominator of $6/\sqrt{3}$

$$6/\sqrt{3}$$

Start with the original fraction

$$6/\sqrt{3} \times \sqrt{3}/\sqrt{3}$$

Multiply top and bottom by $\sqrt{3}$

$$6 \times \sqrt{3} / \sqrt{3} \times \sqrt{3}$$

Multiply numerator and denominator

$$6 \times \sqrt{3} / 3$$

Multiply the roots in the denominator

$$2 \times \sqrt{3}$$

Simplify the fraction

$$2\sqrt{3}$$

answer

Watch out

- **Only multiplying the bottom** Multiply both top and bottom by the same root
- **Not simplifying the final fraction** Divide numbers and simplify the result

Exam-style questions

Show your working: most marks are for the method. F = Foundation / IGCSE Core, H = Higher / IGCSE Extended. Answers at the back.

A • SIMPLIFY

1 Simplify $\sqrt{20}$. H [2]

2 Simplify $\sqrt{45}$. H [2]

3 Simplify $\sqrt{98}$. H [2]

4 Simplify $\sqrt{300}$. H [2]

5 Simplify $\sqrt{128}$. H [2]

B • MULTIPLY AND DIVIDE

6 Simplify $\sqrt{5} \times \sqrt{5}$ H [2]

7 Simplify $2\sqrt{3} \times 4\sqrt{3}$ H [2]

8 Simplify $\sqrt{6} \times \sqrt{10}$ H [2]

9 Simplify $\sqrt{48} \div \sqrt{3}$ H [2]

10 Write $\sqrt{8} \times \sqrt{18}$ as an integer H [2]

C • ADD AND SUBTRACT

11 Simplify $3\sqrt{2} + 5\sqrt{2}$ H [3]

12 Simplify $4\sqrt{3} + 2\sqrt{12}$ H [3]

13 Simplify $7\sqrt{5} - 2\sqrt{20}$ H [3]

14 Simplify $2\sqrt{18} - \sqrt{8} + \sqrt{32}$ H [3]

D • EXPAND BRACKETS

15 Expand and simplify $(\sqrt{2} + 3)(\sqrt{2} - 3)$ H [3]

16 Expand and simplify $(\sqrt{5} + 2)^2$ H [3]

17 Expand and simplify $\sqrt{3}(2 + \sqrt{3})$ H [3]

18 Expand and simplify $(\sqrt{6} + \sqrt{2})^2$ H [3]

E · RATIONALISE

- | | | |
|-----------|---|-----|
| 19 | Rationalise the denominator of $5/\sqrt{5}$ H | [3] |
| <hr/> | | |
| 20 | Rationalise the denominator of $12/\sqrt{6}$ H | [3] |
| <hr/> | | |
| 21 | Rationalise the denominator of $1/(2\sqrt{3})$ H | [3] |
| <hr/> | | |
| 22 | Rationalise the denominator of $4/(3 - \sqrt{5})$ H | [3] |
| <hr/> | | |
| 23 | Show that $\sqrt{75} = 5\sqrt{3}$ H | [3] |
| <hr/> | | |

Marks are a guide to how much working an exam would expect. Questions written by Bloom Education in the style of GCSE (AQA, Edexcel, OCR) and Cambridge IGCSE papers.

Check your work

1 $2\sqrt{5} \sqrt{20} = \sqrt{(4 \times 5)}$. The 4 leaves as 2. So $2\sqrt{5}$.

2 $3\sqrt{5} \sqrt{45} = \sqrt{(9 \times 5)}$. The 9 leaves as 3. So $3\sqrt{5}$.

3 $7\sqrt{2} \sqrt{98} = \sqrt{(49 \times 2)}$. The 49 leaves as 7. So $7\sqrt{2}$.

4 $10\sqrt{3} \sqrt{300} = \sqrt{(100 \times 3)}$. The 100 leaves as 10. So $10\sqrt{3}$.

5 $8\sqrt{2} \sqrt{128} = \sqrt{(64 \times 2)}$. The 64 leaves as 8. So $8\sqrt{2}$.

6 $5 \sqrt{5} \times \sqrt{5} = \sqrt{(5 \times 5)} = \sqrt{25} = 5$

7 $24 \ 2 \times 4 = 8, \sqrt{3} \times \sqrt{3} = \sqrt{9} = 3, 8 \times 3 = 24$

8 $2\sqrt{15} \sqrt{(6 \times 10)} = \sqrt{60} = \sqrt{(4 \times 15)} = 2\sqrt{15}$

9 $4 \sqrt{(48 \div 3)} = \sqrt{16} = 4$

10 $12 \sqrt{(8 \times 18)} = \sqrt{144} = 12$

11 $8\sqrt{2}$ Both terms have $\sqrt{2}$. Add the numbers in front: $3 + 5 = 8$. So $8\sqrt{2}$.

12 $8\sqrt{3} \sqrt{12} = 2\sqrt{3}$, so $2\sqrt{12} = 4\sqrt{3}$. Then $4\sqrt{3} + 4\sqrt{3} = 8\sqrt{3}$.

13 $3\sqrt{5} \sqrt{20} = 2\sqrt{5}$, so $2\sqrt{20} = 4\sqrt{5}$. Then $7\sqrt{5} - 4\sqrt{5} = 3\sqrt{5}$.

14 $8\sqrt{2}$ Simplify each root: $\sqrt{18} = 3\sqrt{2}$, $\sqrt{8} = 2\sqrt{2}$, $\sqrt{32} = 4\sqrt{2}$. Now all terms have $\sqrt{2}$. Add and subtract: $2 \times 3 = 6$, $6 - 2 = 4$, $4 + 4 = 8$. So $8\sqrt{2}$.

15 -7 Multiply using $(a + b)(a - b) = a^2 - b^2$. $\sqrt{2^2 - 3^2} = 2 - 9 = -7$

16 $9 + 4\sqrt{5}$ Use $(a + b)^2 = a^2 + 2ab + b^2$. $(\sqrt{5})^2 + 2 \times \sqrt{5} \times 2 + 2^2 = 5 + 4\sqrt{5} + 4 = 9 + 4\sqrt{5}$

17 $3 + 2\sqrt{3}$ Multiply each term: $\sqrt{3} \times 2 + \sqrt{3} \times \sqrt{3} = 2\sqrt{3} + 3$

18 $8 + 4\sqrt{3}$ $(\sqrt{6})^2 + 2\sqrt{6} \times \sqrt{2} + (\sqrt{2})^2 = 6 + 2\sqrt{12} + 2 = 8 + 4\sqrt{3}$, since $\sqrt{12} = 2\sqrt{3}$.

19 $\sqrt{5}$ Multiply numerator and denominator by $\sqrt{5}$. $5 \times \sqrt{5} = 5\sqrt{5}$. $\sqrt{5} \times \sqrt{5} = 5$. $5\sqrt{5}/5 = \sqrt{5}$.

20 $2\sqrt{6}$ Multiply numerator and denominator by $\sqrt{6}$. $12 \times \sqrt{6} = 12\sqrt{6}$. $\sqrt{6} \times \sqrt{6} = 6$. $12\sqrt{6}/6 = 2\sqrt{6}$.

21 $\sqrt{3}/6$ Multiply numerator and denominator by $\sqrt{3}$. $1 \times \sqrt{3} = \sqrt{3}$. $2\sqrt{3} \times \sqrt{3} = 2 \times 3 = 6$. $\sqrt{3}/6 = \sqrt{3}/6$.

22 $3 + \sqrt{5}$ Multiply numerator and denominator by $(3 + \sqrt{5})$. $(4 \times (3 + \sqrt{5})) = 12 + 4\sqrt{5}$. $(3 - \sqrt{5})(3 + \sqrt{5}) = 9 - 5 = 4$. $(12 + 4\sqrt{5})/4 = 3 + \sqrt{5}$.

23 $5\sqrt{3}$ Split 75 into square and non-square parts. $75 = 25 \times 3$. $\sqrt{(25 \times 3)} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$.

Exam checklist

- Biggest square number first. Always find the largest square that divides the number.
- Multiply roots together. Combine them under one root when multiplying.
- Add only like surds. Only roots with the same number can be added or subtracted.
- Multiply top and bottom when rationalising. Remove the root from the bottom of a fraction.
- Answer in its simplest form. Make sure no square numbers are inside the root.
- Check your work by expanding or substituting. Ensure your simplified form is the same as the original.

Got stuck on a question?

- Find the section it belongs to and go through its worked example again, line by line.
- Put your answer back into the question. If it doesn't work, look for a sign or a step you skipped.
- Still stuck? Message us on Instagram (@bloom_education_international). We read every message.

Mocks coming up? **Your first lesson is free.**

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