

Simultaneous equations, **finally** easy

Learn to solve two unknowns with receipts. Use elimination, multiplication, and substitution. Try exam-style questions with answers.

1 Same sign? Subtract. Different signs? Add.

2 Multiply a whole receipt first

3 When one equation says $y = \dots$

4 One straight line, one curve

5 Practice + answers

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EXAM-STYLE
QUESTIONS

How can you work out two prices when nobody tells you either one?

Simultaneous equations feel hard because there are two unknowns. Use the café receipts analogy to find the prices. Line up the receipts and subtract to find the difference.

The picture to keep in your head: Two café receipts

Two receipts from the same café: 2 coffees and a cake cost £7, 1 coffee and a cake cost £5. Line them up and the only difference is one extra coffee, which must cost the £2 difference. Subtracting the receipts makes the cake vanish; that is elimination. Multiplying a whole equation is buying the whole order twice: every item and the total double.

The method, every time

1. Line up the receipts to see what's different.
2. Subtract receipts with same signs to remove the same items.
3. Multiply a receipt to match the number of items in the other.
4. Add receipts with different signs to cancel out the same items.
5. Find one price first, then use it to find the other.

Words you'll see: **elimination** = Subtracting equations to remove one variable. · **multiplication** = Scaling an equation to match the other. · **substitution** = Using one equation to find the value of a variable. · **unknowns** = Variables you need to find, like coffee and cake prices. · **receipts** = Equations that show how much each item costs together.

Same sign? Subtract. Different signs? Add.

Imagine two café receipts. If the same item appears with the same sign in both, subtract them to cancel it out. This is like removing a charge with a refund. If the signs differ, add them to cancel. This rule helps you find the value of one item at a time.

WORKED EXAMPLE Solve $3x + 2y = 13$ and $5x - 2y = 11$

$$3x + 2y = 13 \quad (1)$$

$$5x - 2y = 11 \quad (2)$$

$$8x = 24$$

$$x = 3$$

$$9 + 2y = 13, \text{ so } y = 2$$

$$5 \times 3 - 2 \times 2 = 11 \quad \checkmark$$

$$x = 3, y = 2$$

Label the receipts

$+2y$ and $-2y$: different signs, so add

$(1) + (2)$: the y terms cancel

Divide by 8

Put $x = 3$ into (1)

Check in (2)

answer

Watch out

- **Subtracting a negative wrongly** Remember subtracting a negative is adding the positive
- **Forgetting to subtract the totals** Always subtract the entire equation, not just one term
- **Stopping after finding x** Always find both variables to complete the solution

Multiply a whole receipt first

Imagine two café receipts. If you want to match the same item, you might need to buy the whole order again. That's multiplying a whole equation. Multiply one or both equations so one letter matches. Multiply every term and the total.

WORKED EXAMPLE Solve $3x + 2y = 12$ and $4x - 5y = -7$

$$3x + 2y = 12 \quad (1)$$

Nothing matches yet

$$4x - 5y = -7 \quad (2)$$

2 and 5 both go into 10

$$15x + 10y = 60$$

(1) $\times$ 5: every term, the total too

$$8x - 10y = -14$$

(2) $\times$ 2

$$23x = 46, \text{ so } x = 2$$

Add: the y terms cancel

$$6 + 2y = 12, \text{ so } y = 3$$

Put $x = 2$ into (1)

$$x = 2, y = 3$$

answer

Watch out

- **Multiplying the letters but not the total** Multiply every term in the equation, including the total
- **Choosing the harder letter to match** Choose the letter that makes the math easier

When one equation says $y = \dots$

Imagine two receipts from the same café. One says $y = 3x - 4$, the other says $2x + y = 11$. Since y is already known in the first receipt, you can swap it into the second. Like swapping the cake for its price. This is substitution. Just plug the expression for y into the other equation and solve.

WORKED EXAMPLE Solve $y = 3x - 4$ and $2x + y = 11$

$$y = 3x - 4$$

Given equation 1

$$2x + (3x - 4) = 11$$

Substitute $y = 3x - 4$ into equation 2

$$5x - 4 = 11$$

Combine like terms

$$5x = 15$$

Add 4 to both sides

$$x = 3$$

Divide both sides by 5

$$y = 3(3) - 4$$

Substitute $x = 3$ back into equation 1

$$y = 5$$

Calculate the value of y

$$x = 3, y = 5$$

answer

Watch out

- **Dropping the brackets so the minus is lost** Keep brackets when substituting y
- **Putting x back into the wrong equation** Always substitute x into the original $y = 3x - 4$

One straight line, one curve

Imagine two receipts: one has a straight line of items, the other a curved line of items. When a line crosses a curve, they share the same x and y values. To find where they meet, substitute the straight line into the curve. This gives you a quadratic equation. Solve it to find the x -values, then use the straight line to find the matching y -values. The rule is: substitute the straight line into the curve, rearrange to $= 0$, factorise, and find the x -values, then find the y -values.

WORKED EXAMPLE Solve $y = x + 3$ and $y = x^2 + 1$

$$y = x + 3$$

$$x + 3 = x^2 + 1$$

$$0 = x^2 - x - 2$$

$$(x - 2)(x + 1) = 0$$

$$x = 2 \text{ or } x = -1$$

$$y = 2 + 3 = 5$$

$$y = -1 + 3 = 2$$

$$x = 2, y = 5 \text{ and } x = -1, y = 2$$

Start with the straight line equation.

Substitute y from the straight line into the curve.

Rearrange to form a quadratic equation.

Factorise the quadratic equation.

Set each factor equal to zero.

Use the straight line to find y when $x = 2$.

Use the straight line to find y when $x = -1$.

answer

Watch out

- **Giving x values without their y values** Always give x and y values as pairs.
- **Expanding $(x + 1)^2$ as $x^2 + 1$** Expand $(x + 1)^2$ as $x^2 + 2x + 1$.

Exam-style questions

Show your working: most marks are for the method. F = Foundation / IGCSE Core, H = Higher / IGCSE Extended. Answers at the back.

A · SAME NUMBER IN FRONT: ADD OR SUBTRACT

- 1 Solve $x + y = 10$ and $x - y = 4$. F [3]

- 2 Solve $3a + b = 11$ and $a + b = 5$. F [3]

- 3 Solve $2x + 3y = 19$ and $2x + y = 9$. F [3]

- 4 Solve $4p + q = 15$ and $4p - 2q = 6$. F [3]

- 5 Solve $5m + 2n = 24$ and $3m - 2n = 8$. F [3]

B · MULTIPLY FIRST

- 6 Solve $2x + y = 11$ and $x + 3y = 13$. F/H [4]

- 7 Solve $3x + 2y = 16$ and $x - y = 2$. F/H [4]

- 8 Solve $5a - 2b = 14$ and $2a + b = 11$. F/H [4]

- 9 Solve $3x + 4y = 25$ and $2x + 3y = 18$. F/H [4]

- 10 Solve $5x - 3y = 1$ and $2x + 5y = 19$. F/H [4]

- 11 Solve $4x + 3y = 5$ and $3x - 2y = 8$. F/H [4]

C · SUBSTITUTION

- 12 Solve $y = 2x + 1$ and $3x + y = 16$. F/H [3]

- 13 Solve $x = 3y - 2$ and $2x + y = 17$. F/H [3]

- 14 Solve $0.5x + y = 7$ and $x - y = 2$. F/H [3]

D · WORD PROBLEMS

- 15 Two adult and three child cinema tickets cost £36. One adult and two child tickets cost £21. Find the cost of each ticket. F/H [4]

- 16 A taxi charges a fixed fee plus a price per mile. A 5-mile journey costs £9.50 and an 8-mile journey costs £14. Find the fixed fee and the price per mile. F/H [4]

- 17 A jar holds 25 coins, all 10p or 20p. Together they are worth £3.70. How many of each coin are there? F/H [4]

- 18** Two numbers add up to 47. Their difference is 13. Find the two numbers. F/H [4]
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- 19** Amara is three times as old as Ben. In 5 years' time she will be twice as old as him. How old are they now? F/H [4]
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E · LINE AND CURVE

- 20** Solve $y = x + 1$ and $y = x^2 - 1$. H [5]
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- 21** Solve $x^2 + y^2 = 25$ and $y = x + 1$. H [5]
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- 22** Solve $y = 2x - 3$ and $xy = 2$. H [5]
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- 23** The lines $y = 2x + 1$ and $y = 7 - x$ cross at the point P. Find the coordinates of P. H [5]
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Marks are a guide to how much working an exam would expect. Questions written by Bloom Education in the style of GCSE (AQA, Edexcel, OCR) and Cambridge IGCSE papers.

Check your work

1 $x = 7, y = 3$ Add: $2x = 14$.

2 $a = 3, b = 2$ Subtract: $2a = 6$.

3 $x = 2, y = 5$ Subtract: $2y = 10$.

4 $p = 3, q = 3$ Subtract: $3q = 9$.

5 $m = 4, n = 2$ Add: $8m = 32$.

6 $x = 4, y = 3$ $(1) \times 3 - (2): 5x = 20$.

7 $x = 4, y = 2$ $(2) \times 2 + (1): 5x = 20$.

8 $a = 4, b = 3$ $(2) \times 2 + (1): 9a = 36$.

9 $x = 3, y = 4$ $(1) \times 2 - (2) \times 3: -y = -4$.

10 $x = 2, y = 3$ $(1) \times 5 + (2) \times 3: 31x = 62$.

11 $x = 2, y = -1$ $(1) \times 2 + (2) \times 3: 17x = 34$.

12 $x = 3, y = 7$ $3x + 2x + 1 = 16$, so $5x = 15$.

13 $x = 7, y = 3$ $2(3y - 2) + y = 17$, so $7y = 21$.

14 $x = 6, y = 4$ Add: $1.5x = 9$.

15 Adult £9, child £6 $2a + 3c = 36, a + 2c = 21; (2) \times 2 - (1): c = 6$.

16 Fee £2, £1.50 a mile $f + 5p = 9.5, f + 8p = 14$; subtract: $3p = 4.5$.

17 $13 \times 10p, 12 \times 20p$ $t + w = 25, 10t + 20w = 370; (1) \times 10$ and subtract: $10w = 120$.

18 30 and 17 $x + y = 47, x - y = 13$; add: $2x = 60$.

19 Ben 5, Amara 15 $a = 3b$ and $a + 5 = 2(b + 5)$, so $3b + 5 = 2b + 10$.

20 $x = 2, y = 3$ and $x = -1, y = 0$ $x^2 - 1 = x + 1$, so $x^2 - x - 2 = 0 = (x - 2)(x + 1)$.

21 $x = 3, y = 4$ and $x = -4, y = -3$ $x^2 + (x + 1)^2 = 25$, so $x^2 + x - 12 = 0 = (x + 4)(x - 3)$.

22 $x = 2, y = 1$ and $x = -0.5, y = -4$ $x(2x - 3) = 2$, so $2x^2 - 3x - 2 = 0 = (2x + 1)(x - 2)$.

23 $P = (2, 5)$ $2x + 1 = 7 - x$, so $3x = 6$.

Exam checklist

- Label equations (1) and (2) to keep track.
- Multiply every term in an equation to match the other.
- Same sign? Subtract. Different signs? Add. Make sure you get the right one.
- Find both unknowns and check in the other equation.
- Quadratics give two pairs of answers, so check both.

Got stuck on a question?

- Find the section it belongs to and go through its worked example again, line by line.
- Put your answer back into the question. If it doesn't work, look for a sign or a step you skipped.
- Still stuck? Message us on Instagram (@bloom_education_international). We read every message.

Mocks coming up? **Your first lesson is free.**

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