

Sequences and the n th term, **linear and quadratic**

Find the n th term using staircases. Worked examples.
Exam-style questions with answers.

1 Step size times n , plus the zeroth term

2 Find any term, or test a number

3 Half the second difference

4 Geometric and Fibonacci-type

5 Practice + answers

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EXAM-STYLE
QUESTIONS

How can you find the 100th number in a sequence without writing out the first 99?

Sequences feel hard because they hide patterns. Think of them as staircases. Each step shows how numbers change.

The picture to keep in your head: A staircase

A linear sequence is a staircase with equal steps: 5, 8, 11, 14 goes up 3 every step, so the n th term is $3n$ plus where the staircase starts. Step back one stair from the first term to find the start (the zeroth term, 2), so the n th term is $3n + 2$. A quadratic sequence is a staircase whose steps grow steadily: the steps themselves go up by the same amount each time (the second difference), and half of that is the number in front of n^2 .

The method, every time

1. Find the step size by subtracting any two terms.
2. Step back one from the first term to find the zeroth term.
3. Write the n th term as step size times n plus the zeroth term.
4. Check by plugging in $n = 1$ and $n = 2$ to see if you get the right numbers.
5. For quadratics, find half the second difference and use it as the n squared term.

Words you'll see: **nth term** = The rule to find any number in a sequence. · **step size** = How much each term increases by. · **zeroth term** = The number before the first term in a staircase. · **second difference** = The change in step size for quadratic sequences. · **quadratic sequence** = A sequence where steps grow by a steady amount.

Step size times n , plus the zeroth term

A linear sequence is like a staircase with equal steps. The step size is the common difference, and the zeroth term is where the staircase starts. The n th term is step size times n plus the zeroth term. Find the step size first, then subtract it from the first term to get the zeroth term.

WORKED EXAMPLE Find the n th term of 5, 8, 11, 14, ...

5, 8, 11, 14, ...

Start with the sequence

$$8 - 5 = 3$$

The step size is 3

$$5 - 3 = 2$$

Step back one: the zeroth term is 2

$$3n + 2$$

Step size $\times n$, plus the zeroth term

$$n = 1: 3 + 2 = 5 \quad \checkmark \quad n = 2: 6 + 2 = 8 \quad \checkmark$$

Check the first two terms

$$3n + 2$$

answer

Watch out

- **Writing $n + 3$ instead of $3n$** Multiply step size by n , not add it
- **Using the first term instead of the zeroth term** Subtract step size from first term to find zeroth term

Find any term, or test a number

A staircase helps you find any step by counting how many steps you climb. To test if a number is in the sequence, imagine climbing to that step and seeing if it lands exactly on a rung. The rule is: substitute n into the n th term to find a term, or set the n th term equal to the number and solve for n .

WORKED EXAMPLE Is 100 a term of the sequence with n th term $3n + 2$?

$$3n + 2 = 100$$

$$3n = 98$$

$$n = 98 \div 3$$

$$n = 32.666\dots$$

n is not a whole number

100 is not a term of the sequence

Set n th term equal to 100

Subtract 2 from both sides

Divide both sides by 3

Calculate the result

Check if n is a whole number

Only whole-number n means the number is in the sequence

100 is not a term ($n = 32.67$ is not a whole number)

answer

Watch out

- **Saying yes when n isn't a whole number** Only whole-number n means the number is in the sequence
- **Substituting the number for n instead of solving** Set the n th term equal to the number and solve for n

Half the second difference

A quadratic sequence is like a staircase where the steps themselves grow by the same amount each time. The second difference shows how fast the steps are growing. The coefficient of n^2 is half the second difference. Once you know that, subtract the n^2 part from each term and find the remaining linear part. Add them together to get the full n th term.

WORKED EXAMPLE Find the n th term of 4, 9, 16, 25, 36, ...

4, 9, 16, 25, 36

Start with the sequence

first differences 5, 7, 9, 11

Subtract each term from the next

second difference 2

The steps grow by 2

$2 \div 2 = 1$, so n^2

Half the second difference

$4 - 1, 9 - 4, 16 - 9 \dots = 3, 5, 7, 9, 11$

Take away n^2 from each term

$3, 5, 7, \dots = 2n + 1$

What is left is linear

$n^2 + 2n + 1$

Add the two parts

$n^2 + 2n + 1$

answer

Watch out

- **Using the whole second difference as the n^2 coefficient** Divide the second difference by 2 to get the n^2 coefficient
- **Subtracting the n^2 part from the wrong term** Subtract n^2 from each term in order, starting from the first term

Geometric and Fibonacci-type

A geometric sequence is like a staircase that grows by multiplying each step. A Fibonacci-type sequence is like a staircase where each step is the sum of the two before. To find the next term, check if you multiply or add the previous terms. The rule is: geometric sequences multiply by the same number each time, Fibonacci-type sequences add the two previous terms.

WORKED EXAMPLE Find the next term of 3, 6, 12, 24, ...

$$6 \div 3 = 2, 12 \div 6 = 2, 24 \div 12 = 2$$

× 2 each time

$$24 \times 2 = 48$$

48

The differences change, the ratio does not

A geometric sequence

Next term

answer

Watch out

- **Looking only at differences when the rule multiplies** Check if the ratio between terms is constant, not the difference.
- **Adding the step to the wrong term** Add the previous two terms, not just one.

Exam-style questions

Show your working: most marks are for the method. F = Foundation / IGCSE Core, H = Higher / IGCSE Extended. Answers at the back.

A • LINEAR NTH TERM

- 1** Find the n th term of 3, 6, 9, 12, 15, ... F [2]

- 2** Find the n th term of 5, 8, 11, 14, 17, ... F [2]

- 3** Find the n th term of 10, 7, 4, 1, -2, ... F [2]

- 4** Find the n th term of 2.5, 5, 7.5, 10, 12.5, ... F [2]

- 5** Find the n th term of -4, -1, 2, 5, 8, ... F [2]

B • USE IT

- 6** Find the 50th term of 2.5, 5, 7.5, 10, ... F/H [2]

- 7** Is 200 a term of -4, -1, 2, 5, 8, ...? Give a reason. F/H [2]

- 8** Find the first term greater than 100 in 4, 11, 18, 25, ... F/H [2]

- 9** Find the 50th term of -4, -1, 2, 5, 8, ... F/H [2]

C • QUADRATIC

- 10** Find the n th term of 2, 8, 18, 32, 50, ... H [3]

- 11** Find the n th term of 4, 11, 22, 37, 56, ... H [3]

- 12** Find the n th term of 4, 13, 28, 49, 76, ... H [3]

- 13** Find the n th term of 3, 6, 11, 18, 27, ... H [3]

- 14** Find the n th term of 5, 10, 17, 26, 37, ... H [3]

D • OTHER SEQUENCES

- 15** Find the term-to-term rule for the sequence: 2, 6, 10, 14, 18. F/H [3]

- 16** Find the missing terms in the Fibonacci-type sequence: 3, 5, ?, ?, ?. F/H [3]

- 17** Find the next term in the sequence: $\sqrt{2}$, $2\sqrt{2}$, $3\sqrt{2}$, $4\sqrt{2}$, ?. F/H [3]

- 18** Find the next term in the sequence: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, ?. F/H [3]

E · PATTERNS

- 19** A matchstick pattern has 5 sticks in pattern 1 and increases by 3 sticks each time. [3]
Find the n th term and the number of sticks in pattern 20. F/H
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- 20** A matchstick pattern has 8 sticks in pattern 1 and increases by 4 sticks each time. [3]
Find the n th term and the number of sticks in pattern 20. F/H
-
- 21** A matchstick pattern has 12 sticks in pattern 1 and increases by 5 sticks each time. [3]
Find the n th term and the number of sticks in pattern 20. F/H
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Marks are a guide to how much working an exam would expect. Questions written by Bloom Education in the style of GCSE (AQA, Edexcel, OCR) and Cambridge IGCSE papers.

Check your work

- 1** $3n$ Step 3, zeroth term 0.
- 2** $3n + 2$ Step 3, zeroth term $5 - 3 = 2$.
- 3** $-3n + 13$ Step -3, zeroth term $10 + 3 = 13$.
- 4** $2.5n$ Step 2.5, zeroth term 0.
- 5** $3n - 7$ Step 3, zeroth term $-4 - 3 = -7$.
- 6** 125 nth term $2.5n$, so 2.5×50 .
- 7** Yes: it is the 69th term nth term $3n - 7 = 200$ gives $n = 69$, a whole number.
- 8** 102 nth term $7n - 3 > 100$ gives $n > 14.7$, so $n = 15$: $7 \times 15 - 3 = 102$.
- 9** 143 $3 \times 50 - 7 = 143$.
- 10** $2n^2$ Second difference 4, so $2n^2$; nothing is left over.
- 11** $2n^2 + n + 1$ Second difference 4, so $2n^2$. Left over: 2, 3, 4, 5, $6 = n + 1$.
- 12** $3n^2 + 1$ Second difference 6, so $3n^2$. Left over: 1, 1, 1, ... = 1.
- 13** $n^2 + 2$ Second difference 2, so n^2 . Left over: 2 each time.
- 14** $n^2 + 2n + 2$ Second difference 2, so n^2 . Left over: 4, 6, 8, ... = $2n + 2$.
- 15** Add 4 Look at the difference between terms. $6 - 2 = 4$, $10 - 6 = 4$, etc. So each term is 4 more than the previous. Rule: add 4.
- 16** 8, 13, 21 Each term is the sum of the two previous terms. $3 + 5 = 8$, $5 + 8 = 13$, $8 + 13 = 21$. Missing terms: 8, 13, 21.
- 17** $5\sqrt{2}$ Each term increases by $\sqrt{2}$. $4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$. Next term is $5\sqrt{2}$.
- 18** $1/32$ Each term is half of the previous. $1/16 \div 2 = 1/32$. Next term is $1/32$.
- 19** nth term: $3n + 2$, pattern 20: 62 Step 1: First term is 5. Step 2: Difference is 3. Step 3: nth term is $3n + 2$. Step 4: Pattern 20 is $3 \times 20 + 2 = 62$.
- 20** nth term: $4n + 4$, pattern 20: 84 Step 1: First term is 8. Step 2: Difference is 4. Step 3: nth term is $4n + 4$. Step 4: Pattern 20 is $4 \times 20 + 4 = 84$.
- 21** nth term: $5n + 7$, pattern 20: 107 Step 1: First term is 12. Step 2: Difference is 5. Step 3: nth term is $5n + 7$. Step 4: Pattern 20 is $5 \times 20 + 7 = 107$.

Exam checklist

- Step size times n
- Zeroth term = first term minus the step
- Check with $n = 1$ and 2
- Quadratic: half the second difference
- Whole-number n means the number is in the sequence

Got stuck on a question?

- Find the section it belongs to and go through its worked example again, line by line.
- Put your answer back into the question. If it doesn't work, look for a sign or a step you skipped.
- Still stuck? Message us on Instagram (@bloom_education_international). We read every message.

Mocks coming up? **Your first lesson is free.**

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