

Recurring decimals to fractions, **the subtraction trick**

Learn to turn repeating decimals into fractions using a song on repeat analogy. Worked examples, exam-style questions with answers.

1 Dots mark the repeating part

2 Shift by one, subtract

3 Shift by the length of the loop

4 When the repeat starts later

5 Practice + answers

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EXAM-STYLE
QUESTIONS

How can 0.272727... going on forever be exactly $\frac{3}{11}$?

Recurring decimals feel tricky because they go on forever. Think of them like a song on repeat. We'll use a simple trick to turn them into exact fractions.

The picture to keep in your head: A song stuck on repeat

A recurring decimal is a song on repeat: 0.272727... plays '27' forever. If you make a copy and shift it along by exactly one loop (multiply by 100 for a two-digit loop), the two copies line up perfectly from then on. Subtract one from the other and the never-ending repeat cancels completely, leaving a whole number you can turn into a fraction.

The method, every time

1. Let x = the decimal. Like a song, it plays forever.
2. Multiply x by 10, 100, or 1000 to shift the repeat one loop.
3. Subtract the original x from the shifted version to cancel the repeat.
4. Simplify the result to get the fraction.
5. Check by converting the fraction back to a decimal.

Words you'll see: **recurring** = A decimal that repeats the same digits forever. · **loop** = The number of digits that repeat in a decimal. · **shift** = Multiply the decimal by 10, 100, or 1000 to move the repeat. · **subtract** = Take away one copy of the decimal from another to cancel the repeat. · **simplify** = Reduce the fraction to its simplest form by dividing numerator and denominator by their highest common factor.

Dots mark the repeating part

A recurring decimal is like a song stuck on repeat. Dots show which part repeats, just like a song loop. Every fraction is a decimal that either ends or repeats. Every repeating decimal is a fraction. Dots over one digit mean that digit repeats. Dots over the first and last digit of a block mean that block repeats.

WORKED EXAMPLE Write $5/11$ as a recurring decimal

$$5 \div 11 = 0.454545\dots$$

Divide 5.0000... by 11

45 repeats forever

The remainders 6, 5, 6, 5... cycle

$0.\overline{45}$

Mark the repeating block

$0.454545\dots$

answer

Watch out

- **Putting dots over every digit of the block** Only dot the first and last digit of the repeating block
- **Rounding the decimal instead of showing it repeats** Write the full repeating pattern, not a rounded version

Shift by one, subtract

A recurring decimal is like a song on repeat. If you shift the decimal by one loop, the repeat cancels out. Multiply by 10 to shift, subtract the original, and solve for x . This gives you the fraction.

WORKED EXAMPLE Write $0.777\dots$ as a fraction in its simplest form

$$x = 0.777\dots$$

Let x be the recurring decimal.

$$10x = 7.777\dots$$

Multiply by 10 to shift one loop.

$$10x - x = 7.777\dots - 0.777\dots$$

Subtract original x to cancel the repeat.

$$9x = 7$$

The repeat cancels, leaving a whole number.

$$x = 7/9$$

Solve for x by dividing both sides by 9.

$$7/9$$

answer

Watch out

- **Multiplying by 100 when only one digit repeats** Multiply by 10 for one-digit repeats.
- **Not simplifying the fraction** Check if numerator and denominator have common factors.

Shift by the length of the loop

A recurring decimal is like a song on repeat. $0.272727\dots$ plays '27' forever. To turn this into a fraction, you need to make a copy of the decimal and shift it along by exactly the length of the repeating part. This makes the repeating parts line up perfectly. Subtracting one copy from the other cancels the repeating part, leaving a whole number you can turn into a fraction.

WORKED EXAMPLE Write $0.272727\dots$ as a fraction in its simplest form

$$x = 0.272727\dots$$

Let x be the decimal we want to convert.

$$100x = 27.272727\dots$$

Multiply by 100 to shift two digits.

$$100x - x = 27.272727\dots - 0.272727\dots$$

Subtract original x from shifted x .

$$99x = 27$$

The repeating parts cancel out.

$$x = 27/99$$

Solve for x by dividing both sides by 99.

$$x = 3/11$$

Simplify the fraction by dividing numerator and denominator by 9.

$$3/11$$

answer

Watch out

- **Multiplying by 10 for a two-digit loop** Multiply by 100 for two repeating digits.
- **Mistakes subtracting the decimals** Align the decimals before subtracting.

When the repeat starts later

A recurring decimal is like a song that starts with a non-repeating part and then repeats. For $0.1666\dots$, the 1 is a single note before the repeating '6' starts. To catch the repeat, you need to shift the decimal so the repeating part lines up. Use two shifts: one to line up the repeating part and another to line up the non-repeating part. Subtract them to cancel the repeat, then turn the result into a fraction. The rule is: shift the decimal to line up the repeat, subtract, and simplify.

WORKED EXAMPLE Write $0.1666\dots$ as a fraction in its simplest form

$$\text{Let } x = 0.1666\dots$$

Start with the decimal as x .

$$10x = 1.666\dots$$

Shift once to move the non-repeating digit.

$$100x = 16.666\dots$$

Shift again to line up the repeating part.

$$100x - 10x = 16.666\dots - 1.666\dots$$

Subtract to cancel the repeating part.

$$90x = 15$$

The repeating part cancels, leaving a whole number.

$$x = 15/90$$

Solve for x by dividing both sides by 90.

$$x = 1/6$$

Simplify the fraction by dividing numerator and denominator by 15.

$$1/6$$

answer

Watch out

- **Subtracting x instead of $10x$** Always subtract the larger shift from the smaller shift.
- **Forgetting the non-repeating digit** Include the non-repeating digit in your first shift.

Exam-style questions

Show your working: most marks are for the method. F = Foundation / IGCSE Core, H = Higher / IGCSE Extended. Answers at the back.

A · FRACTIONS TO DECIMALS

- 1 Write $\frac{2}{9}$ as a decimal. F/H [2]

- 2 Write $\frac{4}{11}$ as a decimal. F/H [2]

- 3 Write $\frac{1}{3}$ as a decimal. F/H [2]

- 4 Write $\frac{5}{6}$ as a decimal. F/H [2]

B · ONE DIGIT

- 5 Write 0.444... as a fraction in its simplest form. H [2]

- 6 Write 0.888... as a fraction in its simplest form. H [2]

- 7 Write 2.333... as a fraction in its simplest form. H [2]

- 8 Write 0.111... as a fraction in its simplest form. H [2]

C · TWO AND THREE DIGITS

- 9 Write 0.363636... as a fraction in its simplest form. H [3]

- 10 Write 0.818181... as a fraction in its simplest form. H [3]

- 11 Write 0.123123... as a fraction in its simplest form. H [3]

- 12 Write 1.454545... as a fraction in its simplest form. H [3]

D · LATE STARTERS

- 13 Write 0.8333... as a fraction in its simplest form. H [3]

- 14 Write 0.41666... as a fraction in its simplest form. H [3]

- 15 Write 0.2454545... as a fraction in its simplest form. H [3]

- 16 Write 0.0666... as a fraction in its simplest form. H [3]

E · PROVE IT

- 17 Prove algebraically that $0.1818... = \frac{2}{11}$ H [3]

- 18** Show that $0.999\ldots = 1$ H [3]
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- 19** Which of $7/20$, $3/7$ and $5/12$ terminate, and why? H [3]
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Marks are a guide to how much working an exam would expect. Questions written by Bloom Education in the style of GCSE (AQA, Edexcel, OCR) and Cambridge IGCSE papers.

Check your work

1 $0.\overline{2}$ $2 \div 9 = 0.2222\dots$ The 2 repeats, so it is $0.\overline{2}$

2 $0.\overline{36}$ $4 \div 11 = 0.363636\dots$ The '36' repeats, so it is $0.\overline{36}$

3 $0.\overline{3}$ $1 \div 3 = 0.3333\dots$ The 3 repeats, so it is $0.\overline{3}$

4 $0.8\overline{3}$ $5 \div 6 = 0.8333\dots$ The 3 repeats, so it is $0.8\overline{3}$

5 $4/9$ Let $x = 0.444\dots$ Multiply by 10: $10x = 4.444\dots$
Subtract: $10x - x = 4.444\dots - 0.444\dots = 4$. So $9x = 4$. $x = 4/9$.

6 $8/9$ Let $x = 0.888\dots$ Multiply by 10: $10x = 8.888\dots$
Subtract: $10x - x = 8.888\dots - 0.888\dots = 8$. So $9x = 8$. $x = 8/9$.

7 $7/3$ Let $x = 2.333\dots$ Multiply by 10: $10x = 23.333\dots$
Subtract: $10x - x = 23.333\dots - 2.333\dots = 21$. So $9x = 21$. $x = 21/9 = 7/3$.

8 $1/9$ Let $x = 0.111\dots$ Multiply by 10: $10x = 1.111\dots$ Subtract:
 $10x - x = 1.111\dots - 0.111\dots = 1$. So $9x = 1$. $x = 1/9$.

9 $4/11$ Let $x = 0.363636\dots$ Multiply by 100: $100x = 36.363636\dots$ Subtract: $100x - x = 36.363636\dots - 0.363636\dots$
 $99x = 36$. $x = 36/99 = 4/11$

10 $9/11$ Let $x = 0.818181\dots$ Multiply by 100: $100x = 81.818181\dots$ Subtract: $100x - x = 81.818181\dots - 0.818181\dots$
 $99x = 81$. $x = 81/99 = 9/11$

11 $41/333$ Let $x = 0.123123\dots$ Multiply by 1000: $1000x = 123.123123\dots$ Subtract: $1000x - x = 123.123123\dots - 0.123123\dots$
 $999x = 123$. $x = 123/999 = 41/333$

12 $16/11$ $1.4545\dots = 1 + 0.4545\dots$; $100y - y = 45$, so $y = 45/99 = 5/11$, and $1 + 5/11 = 16/11$.

13 $5/6$ $100x = 83.33\dots$, $10x = 8.33\dots$; subtract: $90x = 75$, so $x = 75/90 = 5/6$.

14 $5/12$ $1000x = 416.66\dots$, $100x = 41.66\dots$; subtract: $900x = 375$, so $x = 375/900 = 5/12$.

15 $27/110$ Let $x = 0.2454545\dots$ Multiply by 10 to get $10x = 2.454545\dots$ Multiply by 100 to get $1000x = 245.454545\dots$ Subtract: $1000x - 10x = 245.454545\dots - 2.454545\dots = 243$. So $990x = 243$. Divide: $x = 243 \div 990 = 27/110$.

16 $1/15$ Let $x = 0.0666\dots$ Multiply by 10 to get $10x = 0.666\dots$ Multiply by 10 again to get $100x = 6.666\dots$ Subtract: $100x - 10x = 6.666\dots - 0.666\dots = 6$. So $90x = 6$. Divide: $x = 6 \div 90 = 1/15$.

17 $2/11$ Let $x = 0.1818\dots$ Multiply by 100: $100x = 18.1818\dots$ Subtract: $100x - x = 18.1818\dots - 0.1818\dots = 18$. Divide: $99x = 18$. Solve: $x = 18/99 = 2/11$

18 1 Let $x = 0.999\dots$ Multiply by 10: $10x = 9.999\dots$ Subtract: $10x - x = 9.999\dots - 0.999\dots = 9$. Divide: $9x = 9$. Solve: $x = 1$

19 $7/20$ terminates; $3/7$ and $5/12$ recur A fraction in its simplest form terminates only if its denominator has no prime factors except 2 and 5. $20 = 2^2 \times 5$; 7 is prime; $12 = 2^2 \times 3$.

Exam checklist

- Let x = the decimal. Like a song, it plays forever.
- Multiply by 10, 100, or 1000 to shift one loop.
- Line up the decimals and subtract to cancel the repeat.
- Simplify the fraction by dividing numerator and denominator by their HCF.
- Check by converting the fraction back to a decimal.
- If the repeat starts later, use two shifts to line up the decimals.
- For 'prove' questions, write every line clearly.

Got stuck on a question?

- Find the section it belongs to and go through its worked example again, line by line.
- Put your answer back into the question. If it doesn't work, look for a sign or a step you skipped.
- Still stuck? Message us on Instagram (@bloom_education_international). We read every message.

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