

Rearranging formulae, **change the subject**

Learn how to rearrange formulae like unwrapping a present. Worked examples, exam-style questions with answers.

1 Undo in reverse order

2 Clear the fraction first

3 Square to undo a root, root to undo a square

4 The letter appears twice: collect and factorise

5 Practice + answers

20
EXAM-STYLE
QUESTIONS

If $v = u + at$ gives the speed, how do you turn it round to find the time?

Rearranging formulae feels hard because the letter you want is hidden. Think of it like unwrapping a present. You undo steps in reverse order to find the letter you need.

The picture to keep in your head: Unwrapping a present

A formula wraps the letter you want in layers: in $y = 3x + 5$, x was first multiplied by 3, then had 5 added. To get x out you unwrap in reverse order, like taking off your shoes before your socks: undo the $+ 5$ first (subtract 5), then undo the $\times 3$ (divide by 3). Whatever you do to one side, you do to the other, so the formula stays true.

The method, every time

1. Undo in reverse order: Start with the last step done to the letter you want.
2. Do the same to both sides: Whatever you do to one side, do to the other.
3. Clear the fraction first: If there's a fraction, undo it before other steps.
4. Use $\pm$ for square roots: When you undo a square, remember to include both positive and negative answers.
5. Collect and factorise: If the letter appears twice, group and simplify first.

Words you'll see: **subject** = the letter on its own on one side, e.g. t in $t = (v - u)/a$ · **inverse operation** = the step that undoes another: $+$ undoes $-$, $\times$ undoes $\div$, square root undoes square · **reverse order** = undo the last thing done to the letter first · **factorise** = take the letter out as a common factor, e.g. $ax - bx = x(a - b)$

Undo in reverse order

Think of a formula like a wrapped present. The letter you want is hidden under layers. To get it out, you undo each layer in reverse order, like taking off your shoes before your socks. Whatever you do to one side of the formula, you do to the other. This keeps the formula true.

WORKED EXAMPLE Make t the subject of $v = u + at$

$$v = u + at$$

Start with the original formula

$$v - u = at$$

Undo the $+ u$ by subtracting u from both sides

$$(v - u)/a = t$$

Undo the $\times a$ by dividing both sides by a

$$t = (v - u)/a$$

Rewrite to make t the subject

$$t = (v - u)/a$$

answer

Watch out

- **Dividing only part of one side** Divide the whole side, not just part of it
- **Undoing in the wrong order** Undo addition/subtraction first, then multiplication/division

Clear the fraction first

Unwrapping a present is like rearranging a formula. If the formula has a fraction, it's like the present is inside a box. To get x out, you start by clearing the fraction. Multiply both sides by the denominator to remove the fraction. Whatever you do to one side, you do to the other.

WORKED EXAMPLE Make x the subject of $y = (2x - 3)/5$

$$y = (2x - 3)/5$$

Start with the original equation

$$5y = 2x - 3$$

Multiply both sides by 5 to clear the fraction

$$5y + 3 = 2x$$

Add 3 to both sides to move it to the other side

$$x = (5y + 3)/2$$

Divide both sides by 2 to make x the subject

$$x = (5y + 3)/2$$

answer

Watch out

- **Multiplying only the top by 5** Multiply the whole right-hand side by 5
- **Forgetting brackets when writing the answer** Keep brackets around the numerator to show it's all divided by 2

Square to undo a root, root to undo a square

Squaring and square-rooting undo each other. In $A = \pi r^2$, r was squared first, then multiplied by π . Unwrap in reverse order: divide by π , then square root. If the letter is under a root instead, square both sides to take the root off. Whatever you do, do it to the whole of both sides.

WORKED EXAMPLE Make r the subject of $A = \pi r^2$

$$A = \pi r^2$$

Start with the original formula

$$A/\pi = r^2$$

Divide both sides by π to undo the multiplication

$$\sqrt{A/\pi} = r$$

Take the square root of both sides to undo the square

$$r = \sqrt{A/\pi}$$

Write the answer with r on the left

$$r = \sqrt{A/\pi}$$

answer

Watch out

- **Square-rooting only part of a side** Always square-root the whole side, not just part of it.
- **Writing $\sqrt{A}/\pi$** Divide inside the square root first, then take the root.

The letter appears twice: collect and factorise

When the same letter appears twice in a formula, it's like having two parts of the present wrapped together. You need to collect all the letter terms on one side and move everything else to the other. Then, you factorise the letter out and divide both sides by what's in the bracket. This way, you unwrap the letter properly.

WORKED EXAMPLE Make x the subject of $ax + 3 = bx - 7$

$$ax + 3 = bx - 7$$

Start with the original equation

$$ax - bx = -7 - 3$$

Move all x terms to left and numbers to right

$$x(a - b) = -10$$

Factorise x out of the left side

$$x = -10 / (a - b)$$

Divide both sides by $(a - b)$

$$x = 10 / (b - a)$$

Multiply numerator and denominator by -1

$$x = 10 / (b - a)$$

answer

Watch out

- **Trying to divide before collecting the x terms** Always collect like terms first
- **Sign errors when moving terms** Change the sign when moving terms across the equals sign

Exam-style questions

Show your working: most marks are for the method. F = Foundation / IGCSE Core, H = Higher / IGCSE Extended. Answers at the back.

A · ONE AND TWO STEPS

1 Make s the subject of $P = 4s$. F [2]

2 Make x the subject of $y = mx + c$. F [2]

3 Make w the subject of $A = lw$. F [2]

4 Make r the subject of $C = 2\pi r$. F [2]

5 Make u the subject of $v = u + at$. F [2]

B · FRACTIONS AND BRACKETS

6 Make v the subject of $s = (u + v)t/2$. F/H [3]

7 Make C the subject of $F = 9C/5 + 32$. F/H [3]

8 Make x the subject of $y = 3(x - 4)$. F/H [3]

9 Make h the subject of $A = (a + b)h/2$. F/H [3]

C · POWERS AND ROOTS

10 Make c the subject of $E = mc^2$. H [3]

11 Make r the subject of $V = \pi r^2 h$. H [3]

12 Make l the subject of $T = 2\pi\sqrt{l/g}$. H [3]

13 Make x the subject of $y = \sqrt{x + 4}$. H [3]

D · LETTER TWICE

14 Make x the subject of $3x + a = bx$. H [4]

15 Make x the subject of $y = (x + 2)/(x - 1)$. H [4]

16 Make x the subject of $p(x + q) = r(x - q)$. H [4]

17 Make t the subject of $5t - k = 2(t + k)$. H [4]

E · USE IT

- 18** The formula for temperature is $F = 9C/5 + 32$. Find C when $F = 77$. F/H [3]
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- 19** The area of a circle is 50 cm^2 . Rearrange $A = \pi r^2$ to find r , to 1 decimal place. F/H [3]
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- 20** The formula for velocity is $v = u + at$. Find u when $v = 30$, $a = 4$, $t = 5$. F/H [3]
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Marks are a guide to how much working an exam would expect. Questions written by Bloom Education in the style of GCSE (AQA, Edexcel, OCR) and Cambridge IGCSE papers.

Check your work

1 $s = P/4$ Divide both sides by 4.

2 $x = (y - c)/m$ Subtract c , then divide by m .

3 $w = A/l$ Divide both sides by l .

4 $r = C/(2\pi)$ Divide both sides by 2π .

5 $u = v - at$ Subtract at from both sides.

6 $v = 2s/t - u$ Multiply by 2, divide by t , subtract u .

7 $C = 5(F - 32)/9$ Subtract 32, multiply by 5, divide by 9.

8 $x = y/3 + 4$ Divide by 3, then add 4.

9 $h = 2A/(a + b)$ Multiply by 2, then divide by the whole bracket $(a + b)$.

10 $c = \sqrt{(E/m)}$ Divide by m , then square root.

11 $r = \sqrt{(V/(\pi h))}$ Divide by πh , then square root.

12 $l = gT^2/(4\pi^2)$ Divide by 2π , square both sides, multiply by g .

13 $x = y^2 - 4$ Square both sides, then subtract 4.

14 $x = a/(b - 3)$ Collect x terms: $a = bx - 3x = x(b - 3)$.

15 $x = (y + 2)/(y - 1)$ $y(x - 1) = x + 2$, so $xy - x = y + 2$ and $x(y - 1) = y + 2$.

16 $x = q(p + r)/(r - p)$ $px + pq = rx - rq$, so $pq + rq = rx - px = x(r - p)$.

17 $t = k$ $5t - k = 2t + 2k$, so $3t = 3k$.

18 25 Start with $F = 9C/5 + 32$. Subtract 32: $77 - 32 = 45$. Multiply by 5: $45 \times 5 = 225$. Divide by 9: $225 \div 9 = 25$. So $C = 25$.

19 $r = 4.0 \text{ cm}$ $r = \sqrt{(A/\pi)} = \sqrt{(50/\pi)} = 3.989\dots$

20 10 Start with $v = u + at$. Subtract at : $30 - (4 \times 5) = 30 - 20 = 10$. So $u = 10$.

Exam checklist

- Undo in reverse order: Start with the last step done to the letter you want.
- Do the same to both sides: Keep the equation balanced as you work.
- Clear the fraction first: Undo fractions before other operations to avoid mistakes.
- Use $\pm$ for square roots: Remember both positive and negative answers when you square root.
- Collect and factorise: If the letter appears twice, group and simplify first.
- Check by substituting: Plug in numbers to make sure your rearranged formula works.

Got stuck on a question?

- Find the section it belongs to and go through its worked example again, line by line.
- Put your answer back into the question. If it doesn't work, look for a sign or a step you skipped.
- Still stuck? Message us on Instagram (@bloom_education_international). We read every message.

Mocks coming up? **Your first lesson is free.**

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