

Inequalities, solve them like equations, almost

Learn how to solve inequalities with step-by-step examples and exam-style questions with answers.

1 Same steps as an equation

2 Divide by a negative? Flip the sign

3 Open circle, filled circle

4 Find where the curve is above or below zero

5 Practice + answers

20
EXAM-STYLE
QUESTIONS

Why does the inequality sign flip when you divide by a negative number?

Inequalities can feel tricky because they're not just about numbers—they're about balance. Think of them like a tipped see-saw. When you multiply or divide by a negative, the see-saw tips the other way, so you flip the inequality sign.

The picture to keep in your head: A tipped see-saw

An inequality is a see-saw that's tipped: $2x + 1 > 7$ means the left side is heavier. Adding, subtracting, multiplying or dividing both sides by the same positive number keeps it tipped the same way. Multiplying or dividing by a negative number is like turning the see-saw round: the heavy side is now on the other side, so the sign flips. $5 > 2$, but $-5 < -2$.

The method, every time

1. Treat the inequality like an equation. Add or subtract the same number from both sides.
2. Multiply or divide by a positive number. The see-saw stays tipped the same way.
3. Divide by a negative number. Turn the see-saw round and flip the inequality sign.
4. Sketch the number line. Use an open circle for $<$ or $>$, a filled circle for $\leq$ or $\geq$.
5. Check your answer. Substitute a value from the solution set into the original inequality.

Words you'll see: **inequality** = A mathematical statement showing one value is greater than or less than another. · **flip** = To change the direction of an inequality when multiplying or dividing by a negative. · **interval** = A range of values that satisfy an inequality, shown on a number line. · **quadratic** = An equation with a squared term, often solved by finding where it's above or below zero. · **boundary** = The exact point where the inequality changes from true to false or vice versa.

Same steps as an equation

An inequality is like a tipped see-saw. The left side is heavier than the right side. To solve it, do the same to both sides, like balancing the see-saw. The answer is a range of values, not just one number. This is the same as solving an equation, but the result is a range, not a single value.

WORKED EXAMPLE Solve $2x + 1 > 7$

$$2x + 1 > 7$$

Start with the inequality

$$2x > 6$$

Subtract 1 from both sides

$$x > 3$$

Divide both sides by 2

$$x > 3$$

answer

Watch out

- **Writing $x = 3$** Use $x > 3$ to show the range
- **Losing the inequality sign halfway** Keep the inequality sign until the end

Divide by a negative? Flip the sign

An inequality is like a tipped see-saw. When you divide by a negative number, the see-saw flips. This means the inequality sign must flip too. If you divide both sides by a negative number, always reverse the sign. You can avoid this by keeping the x terms positive.

WORKED EXAMPLE Solve $5 - 3x \leq 14$

$$5 - 3x \leq 14$$

Start with the inequality

$$-3x \leq 9$$

Subtract 5 from both sides

$$x \geq -3$$

Divide by -3 , so flip the sign

$$x \geq -3$$

answer

Watch out

- **Forgetting to flip the sign when dividing by a negative** Always flip the sign when dividing by a negative
- **Flipping the sign when adding or subtracting a negative** Only flip when multiplying or dividing by a negative

Open circle, filled circle

An inequality is like a tipped see-saw. When you solve it, you find all the numbers that make it true. On a number line, open circles show numbers that are not included, like $<$ or $>$. Filled circles show numbers that are included, like $\leq$ or $\geq$. For a double inequality, treat all three parts the same. List integer solutions carefully at the ends.

WORKED EXAMPLE List the integers n such that $-3 < 2n + 1 \leq 7$

$$-3 < 2n + 1 \leq 7$$

Start with the double inequality

$$-4 < 2n \leq 6$$

Subtract 1 from all three parts

$$-2 < n \leq 3$$

Divide all three parts by 2

$$-1, 0, 1, 2, 3$$

-2 is not included ($<$), 3 is ($\leq$)

$$-1, 0, 1, 2, 3$$

answer

Watch out

- **Including an end that isn't included** Check if the inequality is strict ($<$ or $>$) before listing numbers
- **Doing a step to only two of the three parts** Apply the same step to all three parts of the double inequality

Find where the curve is above or below zero

The see-saw works for straight lines; with x^2 you need a quick sketch. Solve the equation to find where the U-shaped curve crosses the x -axis: these are the critical values. Between them the curve dips below the axis, outside them it's above. So ' < 0 ' gives one interval between the critical values, and ' > 0 ' gives two parts outside them.

WORKED EXAMPLE Solve $x^2 - x - 6 < 0$

$$x^2 - x - 6 < 0$$

Start with the inequality

$$(x - 3)(x + 2) = 0$$

Factorise to find the critical values

$$x = 3 \text{ or } x = -2$$

Where the curve crosses the x -axis

$$-2 < x < 3$$

The U-shape is below the axis between them

$$x = 0: 0 - 0 - 6 = -6 < 0 \checkmark$$

Check with a value in the interval

$$-2 < x < 3$$

answer

Watch out

- **Square-rooting both sides of $x^2 > 9$ and writing only $x > 3$** Remember to include both $x > 3$ and $x < -3$.
- **Giving the wrong region (inside instead of outside)** Sketch the curve and test values to find where it is below or above zero.

Exam-style questions

Show your working: most marks are for the method. F = Foundation / IGCSE Core, H = Higher / IGCSE Extended. Answers at the back.

A • SOLVE

1 Solve $2x + 3 > 7$ F [2]

2 Solve $3x - 4 < 5$ F [2]

3 Solve $2(x + 1) > 6$ F [2]

4 Solve $4x + 1 < 3x + 5$ F [2]

5 Solve $(2x + 1)/3 < 3$ F [2]

B • NEGATIVES AND FLIPS

6 Solve $7 - 2x > 1$ F/H [2]

7 Solve $-4x \geq 12$ F/H [2]

8 Solve $3 - x < 8$ F/H [2]

9 Solve $2x + 9 \leq 5x$ without dividing by a negative F/H [2]

C • INTEGERS AND DOUBLE INEQUALITIES

10 List the integers x such that $1 < 2x - 1 \leq 7$. F/H [3]

11 Find the largest integer x that satisfies $3x - 2 < 13$. F/H [3]

12 Find all integer solutions of $2 < 3x - 4 < 5$. F/H [3]

13 Find the smallest integer n such that $4n - 3 > 20$. F/H [3]

D • QUADRATIC

14 Solve $x^2 < 16$ H [3]

15 Solve $x^2 - 5x + 4 \geq 0$ H [3]

16 Solve $x^2 + 2x - 15 > 0$ H [3]

17 A rectangular garden has length $(x + 5)$ m and width $(x - 2)$ m. Find the values of x for which the area is greater than 30 m^2 . H [3]

E · CONTEXT

- 18** A phone plan costs £10 per month plus £0.10 per text. A student has £15 per month. How many texts can they send? F/H [3]
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- 19** A taxi charges £3 plus £2 per mile. Jo has £20. What is the greatest whole number of miles she can travel? F/H [3]
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- 20** A bus company charges £2 per ticket. A group of students has £60 to spend. How many tickets can they buy? F/H [3]
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Marks are a guide to how much working an exam would expect. Questions written by Bloom Education in the style of GCSE (AQA, Edexcel, OCR) and Cambridge IGCSE papers.

Check your work

1 $x > 2$ $2x + 3 > 7 \rightarrow 2x > 4 \rightarrow x > 2$. The see-saw stays tipped because we subtracted 3 and divided by 2 (positive number).

2 $x < 3$ $3x - 4 < 5 \rightarrow 3x < 9 \rightarrow x < 3$. The see-saw stays tipped because we added 4 and divided by 3 (positive number).

3 $x > 2$ $2(x + 1) > 6 \rightarrow x + 1 > 3 \rightarrow x > 2$. The see-saw stays tipped because we divided by 2 (positive number).

4 $x < 4$ $4x + 1 < 3x + 5 \rightarrow x + 1 < 5 \rightarrow x < 4$. The see-saw stays tipped because we subtracted $3x$ and 1 (positive numbers).

5 $x < 4$ $(2x + 1)/3 < 3 \rightarrow 2x + 1 < 9 \rightarrow 2x < 8 \rightarrow x < 4$. The see-saw stays tipped because we multiplied by 3 (positive number).

6 $x < 3$ Subtract 7: $-2x > -6$. Divide by -2 and flip: $x < 3$.

7 $x \leq -3$ Divide by -4 and flip the sign.

8 $x > -5$ Subtract 3: $-x < 5$. Multiply by -1 and flip: $x > -5$.

9 $x \geq 3$ Subtract $2x$ from both sides: $9 \leq 3x$, so $3 \leq x$.

10 2, 3, 4 Add 1: $2 < 2x \leq 8$. Divide by 2: $1 < x \leq 4$.

11 4 $3x - 2 < 13$. Add 2: $3x < 15$. Divide by 3: $x < 5$. Largest integer less than 5 is 4.

12 No integer solutions Add 4: $6 < 3x < 9$. Divide by 3: $2 < x < 3$. No whole number lies strictly between 2 and 3.

13 6 $4n > 23$, so $n > 5.75$; the smallest integer is 6.

14 $-4 < x < 4$ $x^2 < 16 \rightarrow x^2 - 16 < 0 \rightarrow (x - 4)(x + 4) < 0$. The inequality holds when $-4 < x < 4$.

15 $x \leq 1$ or $x \geq 4$ $x^2 - 5x + 4 = (x - 1)(x - 4)$. The inequality holds when $x \leq 1$ or $x \geq 4$.

16 $x < -5$ or $x > 3$ $x^2 + 2x - 15 = (x + 5)(x - 3)$. The inequality holds when $x < -5$ or $x > 3$.

17 $x > 5$ $(x + 8)(x - 5) > 0$ gives $x < -8$ or $x > 5$, but the width $x - 2$ must be positive, so $x > 5$.

18 at most 50 texts ($x \leq 50$) Let x = number of texts. $10 + 0.10x \leq 15$. Subtract 10: $0.10x \leq 5$. Divide by 0.10: $x \leq 50$.

19 8 miles $3 + 2m \leq 20$, so $2m \leq 17$ and $m \leq 8.5$; the greatest whole number is 8.

20 at most 30 tickets ($x \leq 30$) Let x = number of tickets. $2x \leq 60$. Divide by 2: $x \leq 30$.

Exam checklist

- Treat it like an equation. Add or subtract the same number from both sides.
- Flip the sign when multiplying or dividing by a negative. The see-saw tips the other way.
- Use an open circle for $<$ or $>$, a filled circle for $\leq$ or $\geq$. Show where the value lies.
- Double inequalities: do all three parts. Solve for each part separately and find the overlap.
- For quadratics: sketch the curve. Find where it's above or below zero to solve the inequality.

Got stuck on a question?

- Find the section it belongs to and go through its worked example again, line by line.
- Put your answer back into the question. If it doesn't work, look for a sign or a step you skipped.
- Still stuck? Message us on Instagram (@bloom_education_international). We read every message.

Mocks coming up? **Your first lesson is free.**

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