

Indices (powers), negative and fractional too

Learn how to multiply and divide powers using the multiplying machine. Worked examples, exam-style questions with answers.

1 Multiply: add the powers. Divide: subtract them.

2 Zero means no presses; negative means divide

3 The bottom of the fraction is a root

4 Same laws, with letters

5 Practice + answers

23
EXAM-STYLE
QUESTIONS

Why is 5^0 equal to 1, and what could $8^{-1/3}$ possibly mean?

Powers look like a list of rules to memorise, but they all come from one picture: a machine that starts at 1 and multiplies by the base each time you press its button. Once you see the machine, zero, negative and fractional powers make sense.

The picture to keep in your head: The multiplying machine

A power is a machine that starts at 1 and multiplies by the base again and again: 2^3 means start at 1 and press 'times 2' three times, so $1 \rightarrow 2 \rightarrow 4 \rightarrow 8$. A power of 0 means you never press the button, so you're still at 1. A negative power means running the machine backwards, dividing instead: $2^{-3} = 1/8$. A fractional power is a fraction of the job: $8^{1/3}$ asks which number, pressed three times, makes 8; that's the cube root, 2.

The method, every time

1. Start at 1 and press the button as many times as the power says.
2. If you multiply powers with the same base, add the number of presses.
3. If you divide powers with the same base, subtract the number of presses.
4. A zero power means you never press the button, so you stay at 1.
5. A negative power means you run the machine backwards, dividing instead of multiplying.

Words you'll see: **base** = The number you multiply repeatedly in a power, like the 2 in 2^3 . • **power** = How many times you press the multiplying machine button, like the 3 in 2^3 . • **exponent** = Another name for the power, like the 3 in 2^3 . • **negative** = Means you run the multiplying machine backwards, dividing instead of multiplying. • **fractional** = Means you do part of the job, like finding the cube root of 8 for $8^{1/3}$.

Multiply: add the powers. Divide: subtract them.

The multiplying machine helps you understand powers. When you multiply numbers with the same base, you're just adding more presses. When you divide, you're undoing some presses. This means you add powers when multiplying and subtract them when dividing. The rule is: same base, add or subtract powers.

WORKED EXAMPLE Simplify $(3^4 \times 3^5) \div 3^6$, giving your answer as a power of 3

$$(3^4 \times 3^5) \div 3^6$$

$$3^4 \times 3^5$$

$$3^{4+5}$$

$$3^9 \div 3^6$$

$$3^{9-6}$$

$$3^3$$

$$3^3$$

Start with the question as written

Multiply first, same base, add powers

Add the powers when multiplying same base

Now divide, same base, subtract powers

Subtract the powers when dividing same base

Simplify the exponent

answer

Watch out

- **Multiplying the powers when multiplying numbers** Add the powers, not multiply them
- **Using the laws on different bases** Only apply the rules when the base is the same

Zero means no presses; negative means divide

The multiplying machine starts at 1 and multiplies by the base each time. If you press the button 0 times, you're still at 1. A negative power means you run the machine backwards, dividing instead. So $2^0 = 1$ and $2^{-1} = 1/2$. The rule is: $a^0 = 1$ and $a^{-n} = 1/a^n$. A negative power never makes the answer negative.

WORKED EXAMPLE Work out 4^{-2}

$$4^{-2}$$

Start with the question

$$4^{-2} = 1/(4^2)$$

Use the rule for negative powers

$$4^2 = 16$$

Multiply 4 by itself

$$1/16$$

Divide 1 by 16

$$1/16$$

answer

Watch out

- **Writing $4^{-2} = -16$** Negative powers mean dividing, not making the answer negative
- **Thinking anything to the power 0 is 0** Any number to the power 0 is 1, not 0

The bottom of the fraction is a root

$8^{1/3}$ asks: which number, put through the machine three times, makes 8? That's the cube root, 2. So in a fractional power the bottom number is a root and the top number is a power. Do the root first to keep the numbers small: $27^{2/3}$ is the cube root of 27, squared.

WORKED EXAMPLE **Work out $27^{2/3}$**

$$27^{2/3}$$

Start with the question

$$27^{1/3} = 3$$

Bottom number 3: cube root first

$$3^2 = 9$$

Top number 2: then square

$$9$$

answer

Watch out

- **Dividing by the bottom number** Take the root first, not the power
- **Doing the power first and getting huge numbers** Always take the root before the power

Same laws, with letters

The multiplying machine works with letters too. When you square $(2x^3)$, you multiply the number and the power. Powers of the same letter add when you multiply. Let's simplify $(2x^3)^2 \times 3x^{-4}$.

WORKED EXAMPLE Simplify $(2x^3)^2 \times 3x^{-4}$

$$(2x^3)^2 \times 3x^{-4}$$

Start with the original expression.

$$2^2 \times x^{3 \times 2} \times 3 \times x^{-4}$$

Square the 2 and multiply the powers of x.

$$4 \times x^6 \times 3 \times x^{-4}$$

Calculate 2 squared and write the new power of x.

$$4 \times 3 \times x^6 \times x^{-4}$$

Multiply the numbers first.

$$12 \times x^{6 + (-4)}$$

Add the powers of x.

$$12 \times x^2$$

Simplify the power of x.

$$12x^2$$

answer

Watch out

- **Forgetting to square the number in $(2x^3)^2$** Always square both the number and the power.
- **Treating $3x^{-4}$ as $(3x)^{-4}$** Only the x is raised to the power, not the 3.

Exam-style questions

Show your working: most marks are for the method. F = Foundation / IGCSE Core, H = Higher / IGCSE Extended. Answers at the back.

A • THE LAWS

1 Simplify $2^3 \times 2^4$. F [2]

2 Simplify $5^8 \div 5^2$. F [2]

3 Simplify $(3^2)^3$. F [2]

4 Simplify $x^5 \times x^2 \div x^3$. F [2]

5 Simplify $4^2 \times 4^3 \div (4^1)^2$. F [2]

B • ZERO AND NEGATIVE

6 Evaluate 7^0 . F/H [2]

7 Evaluate 5^{-1} . F/H [2]

8 Evaluate 2^{-4} . F/H [2]

9 Evaluate $(1/3)^{-2}$. F/H [2]

10 Which is bigger: 10^{-3} or 10^{-2} ? F/H [2]

C • FRACTIONAL

11 Evaluate $49^{1/2}$. H [2]

12 Evaluate $64^{1/3}$. H [2]

13 Evaluate $16^{3/4}$. H [2]

14 Evaluate $8^{-2/3}$. H [2]

15 Evaluate $(9/25)^{3/2}$. H [2]

D • ALGEBRA

16 Simplify $(3a^2b)^3$. H [2]

17 Simplify $(x^{-2})^{-3}$. H [2]

18 Simplify $6x^5 \div 2x^2$. H [2]

19 Simplify $(16x^8)^{1/2}$. H [2]

E · SOLVE AND SHOW

20 Find n when $2^n = 32$. H [3]

21 Solve $3^x = 1/9$. H [3]

22 Write 8^5 as a power of 2. H [3]

23 Show that $4^3 \times 2^{-2} = 16$. H [3]

Marks are a guide to how much working an exam would expect. Questions written by Bloom Education in the style of GCSE (AQA, Edexcel, OCR) and Cambridge IGCSE papers.

Check your work

1 2^7 Multiply powers with same base: add exponents. $3 + 4 = 7$. Answer: 2^7 .

2 5^6 Divide powers with same base: subtract exponents. $8 - 2 = 6$. Answer: 5^6 .

3 3^6 Power of a power: multiply exponents. $2 \times 3 = 6$. Answer: 3^6 .

4 x^4 Multiply and divide powers with same base: add and subtract exponents. $5 + 2 - 3 = 4$. Answer: x^4 .

5 4^3 Multiply and divide powers with same base: add and subtract exponents. $2 + 3 - 2 = 3$. Answer: 4^3 .

6 17^0 means the multiplying machine never starts. So 1. Answer is 1.

7 0.25^{-1} means running the machine backwards once. So $1 \div 5 = 0.2$. Answer is 0.2.

8 0.06252^{-4} means running the machine backwards four times. So $1 \div (2 \times 2 \times 2 \times 2) = 1 \div 16 = 0.0625$. Answer is 0.0625.

9 9 A negative power flips the fraction: $(1/3)^{-2} = 3^2 = 9$.

10 $10^{-2}10^{-3} = 1 \div 1000 = 0.001$. $10^{-2} = 1 \div 100 = 0.01$. 0.01 is bigger. Answer is 10^{-2} .

11 7 The square root of 49 is the number that, when multiplied by itself, gives 49. That number is 7.

12 4 The cube root of 64 is the number that, when multiplied by itself three times, gives 64. That number is 4.

13 8 First find the fourth root of 16, which is 2. Then cube it: $2 \times 2 \times 2 = 8$.

14 $1/4$ First find the cube root of 8, which is 2. Then take the reciprocal and square it: $1/2 \times 1/2 = 1/4$.

15 $27/125$ First find the square root of $9/25$, which is $3/5$. Then cube it: $3/5 \times 3/5 \times 3/5 = 27/125$.

16 $27a^6b^3$ Multiply the power by each part: $3^3 = 27$, $a^{23} = a^6$, $b^3 = b^3$. Combine: $27a^6b^3$.

17 x^6 Multiply the powers: $-2 \times -3 = 6$. So $x^{-2-3} = x^6$.

18 $3x^3$ Divide coefficients: $6 \div 2 = 3$. Subtract powers: $x^{5-2} = x^3$. So $3x^3$.

19 $4x^4$ Take square root of $16 = 4$ and $x^{8/2} = x^4$. So $4x^4$.

20 $52^n = 32$. $2^5 = 32$. So $n = 5$.

21 $-23^x = 1/9$. $1/9 = 3^{-2}$. So $x = -2$.

22 $2^{15}8 = 2^3$. So $8^5 = (2^3)^5 = 2^{3 \times 5} = 2^{15}$.

23 $164^3 = (2^2)^3 = 2^6$. $2^6 \times 2^{-2} = 2^{6-2} = 2^4 = 16$.

Exam checklist

- Check the base is the same before using the laws.
- Add powers when multiplying, subtract when dividing.
- Multiply the powers when raising a power to another power.
- Anything to the power of 0 is always 1.
- Negative powers mean you flip the fraction and make it positive.
- The denominator of a fractional power is the root you take first.

Got stuck on a question?

- Find the section it belongs to and go through its worked example again, line by line.
- Put your answer back into the question. If it doesn't work, look for a sign or a step you skipped.
- Still stuck? Message us on Instagram (@bloom_education_international). We read every message.

Mocks coming up? **Your first lesson is free.**

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