

# Functions: composite and inverse, **machines in a row**

Learn how to use machines on a conveyor belt to work with composite and inverse functions. Worked examples, exam-style questions, and clear steps.

**1**  $f(3)$  means put 3 into the machine

**2** The machine next to  $x$  goes first

**3** Run the belt backwards

**4** Solve with composite and inverse

**5** Practice + answers

**19**  
EXAM-STYLE  
QUESTIONS

## If $f(x) = 2x + 1$ and $g(x) = x^2$ , why is $fg(3)$ not the same as $gf(3)$ ?

Composite and inverse functions can feel confusing because they involve more than one step. Think of them like machines on a conveyor belt. Each function does a job, and the order matters.

### The picture to keep in your head: Machines on a conveyor belt

A function is a machine:  $f(x) = 2x + 1$  takes a number in, doubles it, adds 1, and sends it out.  $f(3)$  means 'put 3 into machine  $f$ '. A composite function  $fg(x)$  is two machines on a belt: the number goes through  $g$  first (the one nearest the  $x$ ), then through  $f$ . An inverse function  $f^{-1}$  runs the belt backwards, undoing each step in reverse order, so it takes the output back to the input.

### The method, every time

1. Put the number into the machine nearest the  $x$  first.
2. Take the output of the first machine and put it into the second machine.
3. To find an inverse function, undo each step in the opposite order.
4. Check by putting a number through the function and then through its inverse.
5. Remember: inverse functions don't mean 1 over the function.

**Words you'll see:** **function** = a rule (machine) that turns each input into one output · **composite function** = two functions in a row; in  $fg(x)$ ,  $g$  acts first · **inverse function** =  $f^{-1}$  undoes  $f$ : it takes the output back to the input · **input and output** = the number that goes in, and the number that comes out

## $f(3)$ means put 3 into the machine

A function is like a machine on a conveyor belt.  $f(3)$  means put 3 into the machine. The rule tells you what to do with the input. To solve  $f(x) = 11$ , you find the input that makes the machine output 11.

**WORKED EXAMPLE**  $f(x) = 3x - 4$ . Find  $f(5)$ , then solve  $f(x) = 11$ .

$$f(5) = 3 \times 5 - 4$$

Replace  $x$  with 5

$$f(5) = 11$$

Work it out

$$3x - 4 = 11$$

$f(x) = 11$ : which input gives 11?

$$3x = 15$$

Add 4 to both sides

$$x = 5$$

Divide both sides by 3

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$$f(5) = 11, x = 5$$

answer

### Watch out

- **Reading  $f(5)$  as  $f \times 5$**   $f(5)$  means put 5 into the machine, not multiply  $f$  by 5.
- **Forgetting brackets when substituting a negative** Use brackets to keep the negative sign with the number.

## The machine next to $x$ goes first

Think of functions like machines on a conveyor belt. When you have  $fg(x)$ , the machine next to  $x$  ( $g$ ) runs first. The number goes through  $g$ , then through  $f$ . Order matters— $fg$  is not the same as  $gf$ . To find  $fg(x)$ , put  $x$  into  $g$ , then put the result into  $f$ .

**WORKED EXAMPLE**  $f(x) = 2x + 1$  and  $g(x) = x^2$ . Find  $fg(3)$  and  $gf(3)$ .

$$g(3) = 3^2 = 9$$

$fg(3)$ : put 3 into  $g$  first

$$f(9) = 2 \times 9 + 1 = 19$$

Then put the result into  $f$

$$f(3) = 2 \times 3 + 1 = 7$$

$gf(3)$ : put 3 into  $f$  first

$$g(7) = 7^2 = 49$$

Then put the result into  $g$

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$$fg(3) = 19, gf(3) = 49$$

answer

### Watch out

- **Doing  $f$  first in  $fg$**   $fg$  means  $f$  after  $g$ —always do  $g$  first
- **Multiplying  $f(x)$  by  $g(x)$**   $fg$  is not  $f(x) \times g(x)$ —it's  $f(g(x))$

## Run the belt backwards

The inverse runs the belt backwards.  $f(x) = (x + 5)/3$  adds 5, then divides by 3, so  $f^{-1}$  must multiply by 3, then take away 5: the steps are undone in reverse order, like taking off shoes before socks. With algebra: write  $y = f(x)$ , make  $x$  the subject, then write the result with  $x$  as the input.

**WORKED EXAMPLE**  $f(x) = (x + 5)/3$ . Find  $f^{-1}(x)$ .

$$y = (x + 5)/3$$

Write  $y = f(x)$

$$3y = x + 5$$

Multiply both sides by 3

$$x = 3y - 5$$

Subtract 5:  $x$  is now the subject

$$f^{-1}(x) = 3x - 5$$

Write it with  $x$  as the input

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$$f^{-1}(x) = 3x - 5$$

answer

### Watch out

- **Thinking  $f^{-1}(x)$  means  $1/f(x)$**  The  $-1$  means "undo", not "one over".
- **Undoing the steps in the wrong order** Undo the last step first.

## Solve with composite and inverse

Think of composite functions as two machines on a belt. You put a number into the first machine, then the result goes into the second. Inverse functions are like running the belt backwards. To solve problems with composite and inverse functions, build the expression for the composite function, set it equal to the value, and solve. Or use the inverse to work backwards from the answer to the input.

**WORKED EXAMPLE**  $f(x) = x - 3$  and  $g(x) = 2x$ . Solve  $gf(x) = f^{-1}(x)$ .

$$gf(x) = g(f(x)) = g(x - 3) = 2(x - 3)$$

Apply  $f$  first, then  $g$ .

$$gf(x) = 2x - 6$$

Multiply out the brackets.

$$f^{-1}(x) = x + 3$$

Inverse of  $f(x) = x - 3$  is  $x + 3$ .

$$2x - 6 = x + 3$$

Set composite function equal to inverse.

$$2x - x = 3 + 6$$

Subtract  $x$  from both sides.

$$x = 9$$

Simplify both sides.

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$$x = 9$$

answer

### Watch out

- **Using the wrong order for  $gf$**  Always apply the inner function first:  $f(x)$  then  $g(x)$ .
- **Writing  $f^{-1}(x) = x - 3$  instead of  $x + 3$**  Inverse of  $x - 3$  is  $x + 3$ . Add 3 to undo subtracting 3.

# Exam-style questions

Show your working: most marks are for the method. F = Foundation / IGCSE Core, H = Higher / IGCSE Extended. Answers at the back.

## A • NOTATION

- 1 Given  $f(x) = 3x - 2$ , find  $f(2)$ . H [2]

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- 2 Given  $f(x) = 2x + 1$ , find  $f(-3)$ . H [2]

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- 3 Given  $f(x) = x^2 + 4$ , find  $f(1/2)$ . H [2]

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- 4 Given  $f(x) = 5x - 3$ , solve  $f(x) = 7$ . H [2]

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- 5 Given  $f(x) = 2x^2 - 5$ , solve  $f(x) = 3$ . H [2]

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## B • COMPOSITE

- 6 Given  $f(x) = 2x + 1$  and  $g(x) = x - 3$ , find  $fg(2)$ . H [3]

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- 7 Given  $f(x) = 3x - 2$  and  $g(x) = x + 4$ , find  $gf(-1)$ . H [3]

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- 8 Given  $f(x) = 4x - 1$ , find  $ff(3)$ . H [3]

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- 9 Given  $f(x) = 2x + 1$  and  $g(x) = x^2 - 4$ , find an expression for  $fg(x)$  and simplify. H [3]

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- 10 Given  $f(x) = 3x - 2$  and  $g(x) = x + 1$ , find an expression for  $gf(x)$  and simplify. H [3]

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## C • INVERSE

- 11 Given  $f(x) = 2x + 1$ , find  $f^{-1}(7)$ . H [3]

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- 12 Given  $f(x) = (2x - 1)/5$ , find  $f^{-1}(3)$ . H [3]

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- 13 Given  $f(x) = 4/(x + 1)$ , find  $f^{-1}(2)$ . H [3]

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- 14 Given  $f(x) = 3x - 2$ , find  $f^{-1}(7)$ . H [3]

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- 15 Given  $f(x) = 5 - 2x$ , find  $f^{-1}(x)$ . H [3]

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## D • MIXED

- 16 Given  $f(x) = 2x + 3$ , show that  $ff(x) = 4x + 9$ . H [4]

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- 17 Find  $x$  when  $f(x) = f^{-1}(x)$  for  $f(x) = 2x + 1$ . H [4]

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- 18 Given  $f(x) = x + 5$  and  $g(x) = 3x$ , solve  $gf(x) = 27$ . H [4]

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**19** Solve  $fg(x) = 10$  for  $f(x) = 3x + 2$  and  $g(x) = 2x - 1$ . H

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[4]

Marks are a guide to how much working an exam would expect. Questions written by Bloom Education in the style of GCSE (AQA, Edexcel, OCR) and Cambridge IGCSE papers.

# Check your work

**1** 4 Put 2 into machine  $f$ :  $3 \times 2 = 6$ ,  $6 - 2 = 4$ . Answer is 4.

**2** -5 Put -3 into machine  $f$ :  $2 \times -3 = -6$ ,  $-6 + 1 = -5$ . Answer is -5.

**3**  $17/4$  Put  $1/2$  into machine  $f$ :  $(1/2)^2 = 1/4$ ,  $1/4 + 4 = 17/4$ . Answer is  $17/4$ .

**4** 2 Put  $x$  into machine  $f$ :  $5x - 3 = 7$ . Add 3:  $5x = 10$ . Divide by 5:  $x = 2$ . Answer is 2.

**5**  $\pm 2$  Put  $x$  into machine  $f$ :  $2x^2 - 5 = 3$ . Add 5:  $2x^2 = 8$ . Divide by 2:  $x^2 = 4$ . Square root:  $x = \pm 2$ . Answer is  $\pm 2$ .

**6** -1 Put 2 into  $g$ :  $2 - 3 = -1$ . Put -1 into  $f$ :  $2 \times (-1) + 1 = -2 + 1 = -1$ .

**7** -1  $f$  first:  $f(-1) = 3 \times (-1) - 2 = -5$ . Then  $g$ :  $g(-5) = -5 + 4 = -1$ .

**8** 43 Put 3 into  $f$ :  $4 \times 3 - 1 = 12 - 1 = 11$ . Put 11 into  $f$ :  $4 \times 11 - 1 = 44 - 1 = 43$ .

**9**  $2x^2 - 7$   $fg(x) = f(g(x)) = 2 \times (x^2 - 4) + 1 = 2x^2 - 8 + 1 = 2x^2 - 7$ .

**10**  $3x - 1$   $gf(x) = g(f(x)) = (3x - 2) + 1 = 3x - 1$ .

**11** 3 Let  $y = 2x + 1$ . Swap  $x$  and  $y$ :  $x = 2y + 1$ . Solve for  $y$ :  $y = (x - 1)/2$ . So  $f^{-1}(x) = (x - 1)/2$ . Substitute  $x = 7$ :  $f^{-1}(7) = (7 - 1)/2 = 3$ .

**12** 8 Let  $y = (2x - 1)/5$ . Swap  $x$  and  $y$ :  $x = (2y - 1)/5$ . Solve for  $y$ :  $y = (5x + 1)/2$ . So  $f^{-1}(x) = (5x + 1)/2$ . Substitute  $x = 3$ :  $f^{-1}(3) = (5 \times 3 + 1)/2 = 8$ .

**13** 1 Let  $y = 4/(x + 1)$ . Swap  $x$  and  $y$ :  $x = 4/(y + 1)$ . Solve for  $y$ :  $y + 1 = 4/x \rightarrow y = (4/x) - 1$ . So  $f^{-1}(x) = (4/x) - 1$ . Substitute  $x = 2$ :  $f^{-1}(2) = (4/2) - 1 = 1$ .

**14** 3 Let  $y = 3x - 2$ . Swap  $x$  and  $y$ :  $x = 3y - 2$ . Solve for  $y$ :  $y = (x + 2)/3$ . So  $f^{-1}(x) = (x + 2)/3$ . Substitute  $x = 7$ :  $f^{-1}(7) = (7 + 2)/3 = 3$ .

**15**  $f^{-1}(x) = (5 - x)/2$   $y = 5 - 2x$ , so  $2x = 5 - y$  and  $x = (5 - y)/2$ .

**16**  $4x + 9$   $f(x) = 2x + 3$ .  $ff(x) = f(2x + 3) = 2(2x + 3) + 3 = 4x + 6 + 3 = 4x + 9$ .

**17** -1  $f(x) = 2x + 1$ .  $f^{-1}(x) = (x - 1)/2$ . Set  $2x + 1 = (x - 1)/2$ . Multiply both sides by 2:  $4x + 2 = x - 1$ . Subtract  $x$ :  $3x + 2 = -1$ . Subtract 2:  $3x = -3$ . Divide:  $x = -1$ .

**18**  $x = 4$   $gf(x) = g(x + 5) = 3(x + 5) = 3x + 15 = 27$ , so  $3x = 12$ .

**19**  $11/6$   $fg(x) = f(g(x)) = f(2x - 1) = 3(2x - 1) + 2 = 6x - 3 + 2 = 6x - 1$ . Set  $6x - 1 = 10$ . Add 1:  $6x = 11$ . Divide:  $x = 11/6$ .

## Exam checklist

- $fg$  means put  $x$  into  $g$  first, then put that result into  $f$ .
- Inverse functions undo each step in reverse order.
- Replace  $x$  with the number in the original function first.
- Check your answer by putting a number through both functions.
- Don't mix up the order of machines or steps.
- Always write the inverse function with a superscript  $-1$ .
- Never write  $f^{-1}$  as  $1/f$  — it's a different job.

### Got stuck on a question?

- Find the section it belongs to and go through its worked example again, line by line.
- Put your answer back into the question. If it doesn't work, look for a sign or a step you skipped.
- Still stuck? Message us on Instagram (@bloom\_education\_international). We read every message.

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