

Factorising quadratics, **without guessing**

Learn how to factorise quadratics using the rectangle of tiles method. Worked examples, exam-style questions with answers.

1 Factorising is making a rectangle

2 When the numbers are negative

3 No middle term? Look for two squares

4 When there's a number in front of x^2

5 Practice + answers

23
EXAM-STYLE
QUESTIONS

Why do two brackets multiply out to $x^2 + 5x + 6$, and how do you get back from one to the other?

Factorising feels like guesswork until you see what it really is: $x^2 + 5x + 6$ is a pile of tiles, and factorising arranges them into one rectangle. The rectangle's two sides are the two brackets.

The picture to keep in your head: A rectangle of tiles

$x^2 + 5x + 6$ is a pile of tiles: one big x -by- x square, five x -by-1 strips and six small 1-by-1 squares. Factorising means arranging all the tiles into one neat rectangle. The rectangle's two side lengths are the two brackets: $(x + 2)$ along one side and $(x + 3)$ along the other. The 6 small squares must fill the corner, so the two numbers multiply to 6; the strips run along the two sides, so the numbers add to 5.

The method, every time

1. Take out any common factor first.
2. Find two numbers that multiply to the last number and add to the middle number.
3. Write them in the brackets: $(x + p)(x + q)$, signs included.
4. No middle term? Look for a difference of two squares. A number in front of x^2 ? Split the middle term.
5. Expand your brackets to check you get back to the question.

Words you'll see: **factorise** = write as brackets multiplied together · **expand** = multiply out the brackets (the opposite of factorising) · **coefficient** = the number in front of a letter, e.g. 5 in $5x$ · **difference of two squares** = a square minus a square, e.g. $x^2 - 49$

Factorising is making a rectangle

Factorising is like building a rectangle from a pile of tiles. When you expand $(x + 2)(x + 3)$, you get $x^2 + 5x + 6$. Factorising is the opposite: taking the pile of tiles and finding the rectangle's sides. The two numbers in the brackets must multiply to the last number and add to the middle number. This helps you arrange the tiles into a neat rectangle.

WORKED EXAMPLE Factorise $x^2 + 5x + 6$

$$x^2 + 5x + 6$$

Start with the question

$$2 \times 3 = 6 \text{ and } 2 + 3 = 5$$

Two numbers: multiply to 6, add to 5

$$(x + 2)(x + 3)$$

Put them in the brackets

$$x^2 + 3x + 2x + 6 = x^2 + 5x + 6$$

Expand to check

$$(x + 2)(x + 3)$$

answer

Watch out

- **Two numbers that multiply to 6 but don't add to 5** Always check the numbers add to the middle term
- **Forgetting to check by expanding** Expand your answer to make sure it matches the original expression

When the numbers are negative

When you factorise a quadratic with a negative constant term, think of it as a rectangle with some tiles missing. The last number is negative, so one number is positive and one is negative. The middle term's sign shows which one is bigger. For $x^2 - 2x - 15$, you need two numbers that multiply to -15 and add to -2 . The bigger number takes the middle term's sign. This helps you find the right pair of numbers.

WORKED EXAMPLE Factorise $x^2 - 2x - 15$

$$x^2 - 2x - 15$$

Start with the question

$$-5 \times 3 = -15 \text{ and } -5 + 3 = -2$$

Last number negative: one of each sign

$$(x - 5)(x + 3)$$

The bigger number takes the middle sign

$$x^2 + 3x - 5x - 15 = x^2 - 2x - 15$$

Expand to check

$$(x - 5)(x + 3)$$

answer

Watch out

- **Getting the signs the wrong way round, e.g. $(x + 5)(x - 3)$** The bigger number takes the middle term's sign.
- **Treating -15 as 15** Always keep the negative sign with the number.

No middle term? Look for two squares

If there's no middle term, the rectangle has only two squares. One square is big, like x^2 , and the other is small, like 49. To factorise, you need to find two squares that can form a rectangle when one is subtracted from the other. The rule is: $x^2 - a^2 = (x + a)(x - a)$.

WORKED EXAMPLE Factorise $x^2 - 49$

$$x^2 - 49$$

Start with the question

$$x^2 - 7^2$$

49 is 7 squared

$$(x + 7)(x - 7)$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^2 - 7x + 7x - 49 = x^2 - 49$$

The middle terms cancel

$$(x + 7)(x - 7)$$

answer

Watch out

- $(x - 7)^2$ Squares of differences are not the same as difference of squares.
- $x^2 + 49$ Sum of squares doesn't factorise with real numbers.

When there's a number in front of x^2

When there's a number in front of x^2 , the tile rectangle gets more squares. The big x^2 square is now $a \times x^2$, so you need $a \times x^2$ tiles. To factorise, find two numbers that multiply to $a \times c$ and add to b . Split the middle term and factorise in pairs. This works like arranging more tiles into a rectangle.

WORKED EXAMPLE Factorise $2x^2 + 7x + 3$

$$2x^2 + 7x + 3$$

Start with the question

$$a \times c = 2 \times 3 = 6$$

Multiply the first and last numbers

$$1 \times 6 = 6 \text{ and } 1 + 6 = 7$$

Two numbers: multiply to 6, add to 7

$$2x^2 + x + 6x + 3$$

Split the middle term

$$x(2x + 1) + 3(2x + 1)$$

Factorise each pair

$$(2x + 1)(x + 3)$$

Take out the common bracket

$$(2x + 1)(x + 3)$$

answer

Watch out

- **Using c instead of $a \times c$** Always multiply a and c first
- **Not taking out a common factor first** Factor out any common factor before splitting
- **Splitting the middle term incorrectly** Check that the two numbers multiply to $a \times c$ and add to b

Exam-style questions

Show your working: most marks are for the method. F = Foundation / IGCSE Core, H = Higher / IGCSE Extended. Answers at the back.

A • TWO POSITIVE NUMBERS

1 Factorise fully: $x^2 + 7x + 12$ F [2]

2 Factorise fully: $x^2 + 8x + 15$ F [2]

3 Factorise fully: $x^2 + 9x + 20$ F [2]

4 Factorise fully: $x^2 + 6x + 8$ F [2]

B • NEGATIVE NUMBERS

5 Factorise fully: $3x^2 - 6x - 24$ F/H [2]

6 Factorise: $x^2 - 5x + 6$ F/H [2]

7 Factorise: $x^2 + 3x - 10$ F/H [2]

8 Factorise: $x^2 - 2x - 15$ F/H [2]

9 Factorise: $x^2 - 7x + 12$ F/H [2]

C • DIFFERENCE OF TWO SQUARES

10 Factorise $4x^2 - 9$ F/H [2]

11 Factorise $x^2 - 81$ F/H [2]

12 Factorise fully $2x^2 - 50$ F/H [2]

13 Which of these expressions can be factorised? $x^2 + 16$ or $x^2 - 16$. Explain why. F/H [2]

D • NUMBER IN FRONT OF x^2

14 Factorise $2x^2 + 5x + 3$ H [3]

15 Factorise $3x^2 + 10x + 8$ H [3]

16 Factorise $2x^2 - x - 6$ H [3]

17 Factorise $5x^2 - 13x - 6$ H [3]

18 Factorise $6x^2 + x - 2$ H [3]

E · USE IT

- | | | |
|-----------|---|-----|
| 19 | Solve $2x^2 + 5x - 3 = 0$. F/H | [3] |
| <hr/> | | |
| 20 | A rectangle has area $x^2 + 9x + 14$. Find expressions for its length and width. F/H | [3] |
| <hr/> | | |
| 21 | Work out $101^2 - 99^2$ without a calculator using difference of two squares. F/H | [3] |
| <hr/> | | |
| 22 | Simplify $(x^2 - 9)/(x + 3)$. F/H | [3] |
| <hr/> | | |
| 23 | Solve $x^2 - x - 20 = 0$. F/H | [3] |
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Marks are a guide to how much working an exam would expect. Questions written by Bloom Education in the style of GCSE (AQA, Edexcel, OCR) and Cambridge IGCSE papers.

Check your work

1 $(x + 3)(x + 4)$ We need two numbers that multiply to 12 and add to 7. These are 3 and 4. So the factorisation is $(x + 3)(x + 4)$.

2 $(x + 3)(x + 5)$ We need two numbers that multiply to 15 and add to 8. These are 3 and 5. So the factorisation is $(x + 3)(x + 5)$.

3 $(x + 4)(x + 5)$ We need two numbers that multiply to 20 and add to 9. These are 4 and 5. So the factorisation is $(x + 4)(x + 5)$.

4 $(x + 2)(x + 4)$ We need two numbers that multiply to 8 and add to 6. These are 2 and 4. So the factorisation is $(x + 2)(x + 4)$.

5 $3(x - 4)(x + 2)$ Factor out 3: $3(x^2 - 2x - 8)$. Find two numbers that multiply to -8 and add to -2: -4 and +2. So $(x - 4)(x + 2)$. Full answer: $3(x - 4)(x + 2)$

6 $(x - 2)(x - 3)$ Look for two numbers that multiply to +6 and add to -5: -2 and -3. So $(x - 2)(x - 3)$

7 $(x + 5)(x - 2)$ Find two numbers that multiply to -10 and add to +3: +5 and -2. So $(x + 5)(x - 2)$

8 $(x - 5)(x + 3)$ Find two numbers that multiply to -15 and add to -2: -5 and +3. So $(x - 5)(x + 3)$

9 $(x - 3)(x - 4)$ Find two numbers that multiply to +12 and add to -7: -3 and -4. So $(x - 3)(x - 4)$

10 $(2x + 3)(2x - 3)$ $4x^2 = (2x)^2$ and $9 = 3^2$, so $(2x + 3)(2x - 3)$.

11 $(x + 9)(x - 9)$ $x^2 - 81$ is a difference of two squares. The square root of x^2 is x . The square root of 81 is 9. So the factors are $(x + 9)(x - 9)$.

12 $2(x + 5)(x - 5)$ $2x^2 - 50$ has a common factor of 2. Factor out 2: $2(x^2 - 25)$. $x^2 - 25$ is a difference of two squares. So the full factorisation is $2(x + 5)(x - 5)$.

13 $x^2 - 16$ can be factorised because it is a difference of two squares. $x^2 + 16$ cannot be factorised. $x^2 + 16$ cannot be factorised because it is not a difference of two squares. $x^2 - 16$ is a difference of two squares and can be factorised as $(x + 4)(x - 4)$.

14 $(2x + 3)(x + 1)$ $a \times c = 6$: 2 and 3 add to 5. $2x^2 + 2x + 3x + 3$.

15 $(3x + 4)(x + 2)$ $a \times c = 24$: 6 and 4 add to 10. $3x^2 + 6x + 4x + 8$.

16 $(2x + 3)(x - 2)$ $a \times c = -12$: -4 and 3 add to -1. $2x^2 - 4x + 3x - 6$.

17 $(5x + 2)(x - 3)$ $a \times c = -30$: -15 and 2 add to -13. $5x^2 - 15x + 2x - 6$.

18 $(3x + 2)(2x - 1)$ $a \times c = -12$: 4 and -3 add to 1. $6x^2 + 4x - 3x - 2$.

19 $x = 0.5$ or $x = -3$ $(2x - 1)(x + 3) = 0$, so $2x - 1 = 0$ or $x + 3 = 0$.

20 $(x + 2)$ and $(x + 7)$ $2 \times 7 = 14$ and $2 + 7 = 9$, so $x^2 + 9x + 14 = (x + 2)(x + 7)$.

21 400 Use $(a^2 - b^2) = (a - b)(a + b)$. $(101 - 99)(101 + 99) = 2 \times 200 = 400$.

22 $x - 3$ Factorise numerator to $(x - 3)(x + 3)$. Cancel $(x + 3)$. Answer is $(x - 3)$.

23 $x = 5$ or $x = -4$ $(x - 5)(x + 4) = 0$, so $x = 5$ or $x = -4$.

Exam checklist

- Look for a common factor first and take it out.
- Last number positive: both signs match the middle term. Negative: one of each.
- Expand your answer to check it.
- No middle term: difference of two squares, $(x + a)(x - a)$.
- A sum of squares like $x^2 + 16$ does not factorise.
- Number in front of x^2 : find two numbers that multiply to $a \times c$ and add to b .

Got stuck on a question?

- Find the section it belongs to and go through its worked example again, line by line.
- Put your answer back into the question. If it doesn't work, look for a sign or a step you skipped.
- Still stuck? Message us on Instagram (@bloom_education_international). We read every message.

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