

Direct and inverse proportion, **find k, then use it**

Learn how to spot direct and inverse proportion using pizza slices and shared bills. Worked examples, exam-style questions with answers.

1 Double one, double the other

2 Double one, halve the other

3 Proportional to x^2 , or to the square root

4 Tables and graphs

5 Practice + answers

21
EXAM-STYLE
QUESTIONS

If 4 friends share a pizza bill and 8 friends share it instead, why does each person pay half, not double?

Proportion questions are about how two amounts change together. Picture a pizza party: more guests need more pizza (direct), but more guests sharing one bill each pay less (inverse). Every question is the same three moves: find k , write the formula, use it.

The picture to keep in your head: A pizza party

Direct proportion: each guest gets 2 slices, so more guests means more slices, in step: double the guests, double the slices (slices = $2 \times$ guests). Inverse proportion: one pizza bill is shared, so more guests means each pays less: double the guests, half each (each pays = bill $\div$ guests). The fixed number (2 slices each, or the bill) is k , the constant of proportionality. Every question is: find k , write the formula, use it.

The method, every time

1. Decide: direct (both grow together) or inverse (one grows as the other shrinks)?
2. Write the formula with k : $y = kx$, $y = k/x$, $y = kx^2$, and so on.
3. Put in the pair of values you are given to find k .
4. Write the formula again with the number in place of k .
5. Use it to find the value the question asks for, x or y .

Words you'll see: **direct proportion** = both amounts grow together: double one, double the other · **inverse proportion** = one grows as the other shrinks: double one, halve the other · **k** = the constant of proportionality, the fixed number in the formula · **formula** = a rule linking the two amounts, e.g. $y = 4.5x$

Double one, double the other

At a pizza party, if each guest gets 2 slices, more guests mean more slices. This is direct proportion: $y = kx$. Find k with one pair, then use the formula for any other value.

WORKED EXAMPLE y is directly proportional to x . When $x = 4$, $y = 18$. Find y when $x = 10$.

$$y = kx$$

Direct proportion formula

$$18 = k \times 4$$

Substitute $x = 4$, $y = 18$

$$k = 18 \div 4$$

Solve for k

$$k = 4.5$$

Calculate k

$$y = 4.5 \times 10$$

Use the formula with $x = 10$

$$y = 45$$

Multiply to find y

$$45$$

answer

Watch out

- **Adding the difference instead of multiplying** Use multiplication, not addition, to find y
- **Forgetting to write the formula with k** Always write $y = kx$ before using it

Double one, halve the other

At a pizza party, if more guests share the same bill, each pays less. This is inverse proportion. When y is inversely proportional to x , their product is a fixed number k . So, to find k , multiply x and y . Then use k to find the new y when x changes.

WORKED EXAMPLE y is inversely proportional to x . When $x = 3$, $y = 20$. Find y when $x = 12$.

$$y = k/x$$

Inverse proportion formula

$$k = x \times y = 3 \times 20$$

Find k by multiplying x and y

$$k = 60$$

Calculate the constant of proportionality

$$y = 60 / 12$$

Use k to find new y

$$y = 5$$

Divide k by new x to find y

5

answer

Watch out

- **Using $y = kx$ for an inverse question** Use $y = k/x$ for inverse proportion
- **Writing $k = y/x$ instead of $k = xy$** Multiply x and y to find k , not divide

Proportional to x^2 , or to the square root

Some amounts grow faster than in step. A pizza twice as wide has four times the area, because area depends on the width squared. So read the wording carefully: 'proportional to the square of x ' means $y = kx^2$, and 'inversely proportional to the square root of x ' means $y = k/\sqrt{x}$. The method never changes: find k from the pair you're given, write the formula, then use it.

WORKED EXAMPLE y is directly proportional to x^2 . When $x = 3$, $y = 36$. Find x when $y = 100$ (x is positive).

$$y = kx^2$$

y is proportional to x squared

$$36 = k \times 3^2 = 9k$$

Put in $x = 3$, $y = 36$

$$k = 4$$

Divide both sides by 9

$$y = 4x^2$$

Write the formula with k

$$100 = 4x^2$$

Put in $y = 100$

$$x^2 = 25$$

Divide both sides by 4

$$x = 5$$

Square root; x is positive

5

answer

Watch out

- **Forgetting to square x when finding k** Always square x before dividing y by x^2 to find k
- **Squaring y instead of x** Use the correct variable in the formula: $y = kx^2$, not $y^2 = kx$

Tables and graphs

At a pizza party, if more guests come, more slices are needed for direct proportion. If more guests share the same pizza, each pays less for inverse proportion. In a table, direct proportion has $y \div x$ constant; inverse has $x \times y$ constant. Graphs show direct as a straight line through (0, 0), and inverse as a curve that never touches the axes.

WORKED EXAMPLE x : 2, 4, 5 and y : 30, 15, 12. Is y directly or inversely proportional to x ?

$$x = [2, 4, 5], y = [30, 15, 12]$$

$$y \div x = 30 \div 2, 15 \div 4, 12 \div 5$$

$$y \div x = 15, 3.75, 2.4$$

$$x \times y = 2 \times 30, 4 \times 15, 5 \times 12$$

$$x \times y = 60, 60, 60$$

$$k = 60$$

$$y = 60 \div x$$

y is inversely proportional to x

Write down the values from the question

Calculate y divided by x for each pair

Do the division for each pair

Calculate x multiplied by y for each pair

Do the multiplication for each pair

Find the constant k for inverse proportion

Write the formula for inverse proportion

answer

Watch out

- **Saying any straight line is direct proportion** Check if the line passes through (0, 0)
- **Checking only one pair of values** Check all pairs to confirm the constant

Exam-style questions

Show your working: most marks are for the method. F = Foundation / IGCSE Core, H = Higher / IGCSE Extended. Answers at the back.

A · DIRECT, REAL LIFE

- 1 A recipe for 4 cakes needs 200g of flour. How much flour is needed for 10 cakes? F [2]
- 2 5 pens cost £3.40. How much do 8 pens cost? F [2]
- 3 An exchange rate is £1 = €1.25. How many euros do you get for £20? F [2]
- 4 A car uses 6 litres of fuel for 120 km. How much fuel is needed for 200 km? F [2]

B · $Y = KX$ AND $Y = K/X$

- 5 Each guest at a party gets 4 slices of pizza. Write a formula for the total number of slices, s , for g guests, and find s when there are 10 guests. F/H [3]
- 6 y is directly proportional to x . When $x = 6$, $y = 15$. Find y when $x = 10$. F/H [3]
- 7 A £20 pizza bill is shared equally. Write a formula for the cost per person, c , when there are n people, and find c for 8 people. F/H [3]
- 8 y is inversely proportional to x . When $x = 4$, $y = 9$. Find x when $y = 6$. F/H [3]
- 9 A £100 bill is shared equally by 8 people. How much would each person pay if only 5 people shared it? F/H [3]

C · SQUARES, CUBES AND ROOTS

- 10 y is directly proportional to x squared. When $x = 3$, $y = 27$. Write the formula for y in terms of x and find y when $x = 5$. H [4]
- 11 y is inversely proportional to x squared. When $x = 2$, $y = 5$. Write the formula for y in terms of x and find y when $x = 4$. H [4]
- 12 y is directly proportional to the square root of x . When $x = 16$, $y = 8$. Write the formula for y in terms of x and find y when $x = 25$. H [4]
- 13 y is inversely proportional to x cubed. When $x = 2$, $y = 1$. Write the formula for y in terms of x and find y when $x = 1$. H [4]
- 14 y is directly proportional to x cubed. When $x = 2$, $y = 32$. Write the formula for y in terms of x and find x when $y = 128$. H [4]

D · WORKERS AND TIME

- 15** A group of 6 painters take 10 days to paint a house. How long would it take 4 painters to paint the same house? F/H [3]
-
- 16** A journey takes 3 hours at 60 km/h. How long does the same journey take at 90 km/h? F/H [3]
-
- 17** 8 taps fill a tank in 45 minutes. How long would 6 taps take? F/H [3]
-

E · SPOT AND EXPLAIN

- 18** x : 1, 2, 5 and y : 7, 14, 35. Is y directly proportional, inversely proportional, or neither? Give a reason. H [2]
-
- 19** x : 1, 2, 4 and y : 12, 6, 2. Is y inversely proportional to x ? Give a reason. H [2]
-
- 20** y is inversely proportional to x^2 . What happens to y when x is doubled? H [2]
-
- 21** Which of these shows direct proportion: $y = 3x$, $y = 3x + 2$ or $y = 3/x$? Explain. H [2]
-

Marks are a guide to how much working an exam would expect. Questions written by Bloom Education in the style of GCSE (AQA, Edexcel, OCR) and Cambridge IGCSE papers.

Check your work

1 500g 4 cakes = 200g flour, so 1 cake = 50g. 10 cakes = $50 \times 10 = 500\text{g}$. Formula: flour = $50 \times$ cakes.

2 £5.44 5 pens = £3.40, so 1 pen = £0.68. 8 pens = $0.68 \times 8 = £5.44$. Formula: cost = $0.68 \times$ pens.

3 €25 £1 = €1.25, so £20 = $1.25 \times 20 = €25$. Formula: euros = $1.25 \times$ pounds.

4 10 litres 120 km = 6 litres, so 1 km = 0.05 litres. 200 km = $0.05 \times 200 = 10$ litres. Formula: fuel = $0.05 \times$ distance.

5 $s = 4g$, 40 slices Direct proportion with $k = 4$: $s = 4g$, so $s = 4 \times 10 = 40$.

6 25 $k = 15 \div 6 = 2.5$, so $y = 2.5x$ and $y = 2.5 \times 10 = 25$.

7 $c = 20/n$, £2.50 Inverse proportion with $k = 20$: $c = 20/n = 20 \div 8 = 2.50$.

8 $x = 6$ $k = 4 \times 9 = 36$, so $y = 36/x$; $6 = 36/x$ gives $x = 6$.

9 £20 $k = 100$: $100 \div 5 = £20$ each (it was £12.50 each for 8).

10 $y = 3x^2$, $y = 75$ $y = kx^2$. $27 = k \times 3^2 \rightarrow k = 27 \div 9 = 3$. $y = 3x^2$. $y = 3 \times 5^2 = 3 \times 25 = 75$.

11 $y = 20/x^2$, $y = 1.25$ $y = k/x^2$. $5 = k/2^2 \rightarrow k = 5 \times 4 = 20$. $y = 20/x^2$. $y = 20/4^2 = 20/16 = 1.25$.

12 $y = 2\sqrt{x}$, $y = 10$ $y = k\sqrt{x}$. $8 = k \times \sqrt{16} \rightarrow k = 8 \div 4 = 2$. $y = 2\sqrt{x}$. $y = 2 \times \sqrt{25} = 2 \times 5 = 10$.

13 $y = 8/x^3$, $y = 8$ $y = k/x^3$. $1 = k/2^3 \rightarrow k = 1 \times 8 = 8$. $y = 8/x^3$. $y = 8/1^3 = 8$.

14 $y = 4x^3$, $x \approx 3.17$ $y = kx^3$. $32 = k \times 2^3 \rightarrow k = 32 \div 8 = 4$. $y = 4x^3$. $128 = 4x^3 \rightarrow x^3 = 32 \rightarrow x = \sqrt[3]{32} \approx 3.1748$.

15 15 days More painters mean less time. So time = $k \div$ painters. $10 = k \div 6 \rightarrow k = 60$. Time = $60 \div 4 = 15$ days.

16 2 hours Time is inversely proportional to speed: $k = 3 \times 60 = 180$, time = $180 \div 90 = 2$.

17 60 minutes More taps, less time: $k = 8 \times 45 = 360$, time = $360 \div 6 = 60$.

18 directly proportional: $y \div x = 7$ every time $7 \div 1 = 14 \div 2 = 35 \div 5 = 7$, so $y = 7x$.

19 No: $x \times y$ is 12, 12 and 8, not the same each time
For inverse proportion $x \times y$ must stay the same; $4 \times 2 = 8$ breaks it.

20 y is divided by 4 $y = k/x^2$; doubling x multiplies x^2 by 4, so y becomes a quarter of what it was.

21 $y = 3x$: a straight line through (0, 0) $y = 3x + 2$ is straight but misses (0, 0); $y = 3/x$ is inverse.

Exam checklist

- Write the formula with k first, then substitute values.
- Find k from the given pair of numbers.
- Direct: divide y by x to find k . Inverse: multiply x and y to find k .
- Read 'square' and 'square root' carefully in the question.
- Answer the question asked—make sure you find x or y , not just k .

Got stuck on a question?

- Find the section it belongs to and go through its worked example again, line by line.
- Put your answer back into the question. If it doesn't work, look for a sign or a step you skipped.
- Still stuck? Message us on Instagram (@bloom_education_international). We read every message.

Mocks coming up? **Your first lesson is free.**

1-to-1 online tutoring for GCSE, IGCSE and A level, with tutors who explain the why behind every topic.

bloomeducationonline.com