

Algebraic fractions, just fractions with letters

Factorise, cancel, multiply, add, check. Worked examples and exam-style questions with answers.

1 Factorise first, then cancel whole brackets

2 Tops times tops, bottoms times bottoms

3 Same-size slices first

4 Multiply everything by the common denominator

5 Practice + answers

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EXAM-STYLE
QUESTIONS

Why can you cancel $6/8$ down to $3/4$, but not cancel the x 's in $(x + 2)/(x + 5)$?

Algebraic fractions look scary because of the letters, but they follow exactly the same rules as ordinary fractions. Think of pizza slices: you can only add slices of the same size, and you can only cancel when the top and bottom share a whole factor.

The picture to keep in your head: Pizza slices

A fraction is slices of a pizza: the bottom says how many equal slices the pizza is cut into, the top says how many you have. You can only add slices of the same size, so you cut both pizzas into a common number of slices first. Cancelling is just re-cutting the same pizza into bigger slices, which only works when the top and bottom share a whole factor, like 2×3 over 2×4 . Letters follow exactly the same rules as numbers.

The method, every time

1. Factorise the top and the bottom fully first.
2. Cancel only whole brackets (factors) that appear on both the top and the bottom.
3. To multiply, multiply the tops and the bottoms; to divide, flip the second fraction and multiply.
4. To add or subtract, rewrite both fractions over a common denominator, then combine the tops.
5. Check by putting a number in for x in the question and in your answer.

Words you'll see: **numerator** = the top of a fraction · **denominator** = the bottom of a fraction · **factorise** = write as brackets multiplied together, e.g. $x^2 - 9 = (x - 3)(x + 3)$ · **cancel** = divide the top and bottom by the same whole factor · **common denominator** = a bottom both fractions can be rewritten over

Factorise first, then cancel whole brackets

$6/8$ cancels to $3/4$ because $6 = 2 \times 3$ and $8 = 2 \times 4$ share a whole factor of 2: you re-cut the pizza into bigger slices. Algebraic fractions work the same way. Factorise the top and the bottom, then cancel a bracket only if it appears in both. You can't cancel the x 's in $(x + 2)/(x + 5)$: they're joined to other numbers by $+$, so they aren't factors.

WORKED EXAMPLE Simplify $(x^2 - 9)/(x^2 + 5x + 6)$

$$(x^2 - 9)/(x^2 + 5x + 6)$$

Start with the question

$$x^2 - 9 = (x - 3)(x + 3)$$

Top: difference of two squares

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

Bottom: numbers multiply to 6 and add to 5

$$(x - 3)(x + 3) / ((x + 2)(x + 3))$$

$(x + 3)$ is a whole factor of both

$$(x - 3)/(x + 2)$$

Cancel the matching bracket

$$(x - 3)/(x + 2)$$

answer

Watch out

- **Cancelling single terms across a + sign** Factorise first, then cancel whole brackets.
- **Not factorising fully before cancelling** Always factorise both numerator and denominator completely.

Tops times tops, bottoms times bottoms

Multiplying algebraic fractions is like combining pizza slices. Multiply the top slices and bottom slices. Dividing is like sharing a pizza – flip the second pizza and then multiply. Always simplify before multiplying to keep numbers small.

WORKED EXAMPLE Simplify $(2x/(x+1)) \div (4x^2/(3x+3))$

$$(2x/(x+1)) \div (4x^2/(3x+3))$$

Start with the question

$$3x+3 = 3(x+1)$$

Factorise the bottom of the second fraction

$$(2x/(x+1)) \times (3(x+1)/(4x^2))$$

Keep the first, change $\div$ to $\times$, flip the second

$$6x(x+1)/(4x^2(x+1))$$

Tops times tops, bottoms times bottoms

$$6/(4x)$$

Cancel x and $(x+1)$ from top and bottom

$$3/(2x)$$

Divide top and bottom by 2

$$3/(2x)$$

answer

Watch out

- **Flipping the first fraction instead of the second** Always flip the second fraction when dividing
- **Expanding everything instead of factorising** Always factorise first to cancel common terms

Same-size slices first

Imagine two pizzas cut into different numbers of slices. To add or subtract slices, you need to cut both pizzas into the same number of slices first. This is like finding a common denominator. The rule is: to add or subtract fractions, find a common denominator, rewrite each fraction over it, then add or subtract the tops. When subtracting, put brackets around the second top to keep the sign correct.

WORKED EXAMPLE Write $3/(x+2) - 2/(x-1)$ as a single fraction

$$3/(x+2) - 2/(x-1)$$

Start with the question

$$3(x-1)/((x+2)(x-1)) - 2(x+2)/((x+2)(x-1))$$

Same-size slices: both over $(x+2)(x-1)$

$$(3(x-1) - 2(x+2))/((x+2)(x-1))$$

One fraction; brackets round the second top

$$(3x - 3 - 2x - 4)/((x+2)(x-1))$$

Expand; the minus changes both signs

$$(x - 7)/((x+2)(x-1))$$

Collect like terms on the top

$$(x - 7)/((x+2)(x-1))$$

answer

Watch out

- **Adding the bottoms together** Don't add denominators, find a common one instead
- **Losing the minus across the second top** Put brackets around the second top when subtracting

Multiply everything by the common denominator

Imagine each fraction is a pizza slice. To solve an equation with fractions, you multiply everything by the same number of slices (the common denominator) to make the fractions disappear. This lets you solve the equation like a normal one. Multiply every term by the common denominator.

WORKED EXAMPLE Solve $6/x + 6/(x+1) = 5$

$$6/x + 6/(x+1) = 5$$

Start with the equation

$$6(x+1) + 6x = 5x(x+1)$$

Multiply every term by $x(x+1)$

$$12x + 6 = 5x^2 + 5x$$

Expand and collect

$$5x^2 - 7x - 6 = 0$$

Everything on one side

$$(5x+3)(x-2) = 0$$

Factorise

$$x = 2 \text{ or } x = -0.6$$

Neither makes a denominator zero

$$x = 2 \text{ or } x = -0.6$$

answer

Watch out

- **Forgetting to multiply the right-hand side** Multiply every term, not just the left side
- **Keeping a solution that makes a bottom zero** Check that the solution doesn't make any denominator zero

Exam-style questions

Show your working: most marks are for the method. F = Foundation / IGCSE Core, H = Higher / IGCSE Extended. Answers at the back.

A • SIMPLIFY

1 Simplify $(6x + 12)/(3x + 6)$. H [3]

2 Simplify $(x^2 - 9)/(x - 3)$. H [3]

3 Simplify $(x^2 - 16)/(x^2 + 2x - 8)$. H [3]

4 Simplify $(x^2 + 5x + 6)/(x^2 + 4x + 3)$. H [3]

5 Simplify $(x^2 - 2x - 15)/(x^2 - 6x + 5)$. H [3]

B • MULTIPLY AND DIVIDE

6 Simplify $(2x/3) \times (9/(4x))$. H [3]

7 Simplify $(6x/5) \div (3/(10x))$. H [3]

8 Simplify $(x/2) \times (4/x^2)$. H [3]

9 Simplify $(8x/3) \div (2x/9)$. H [3]

10 Simplify $((x + 3)/(x - 2)) \times ((x^2 - 4)/(x^2 + 3x))$. H [3]

C • ADD AND SUBTRACT

11 Simplify $(x/3) + (x/4)$. H [3]

12 Simplify $(x + 1)/3 + (x - 2)/4$. H [3]

13 Simplify $(x + 2)/4 - (x - 1)/3$. H [3]

14 Write $2/(x + 1) + 3/(x - 2)$ as a single fraction. H [3]

15 Write $4/x - 3/(x + 2)$ as a single fraction. H [3]

D • SOLVE

16 Solve $2/x + 3/(x + 1) = 2$. H [4]

17 Solve $(x + 3)/(x - 1) = 2$ H [4]

18 Solve $(x + 2)/(x - 4) = 3$ H [4]

19 Solve $(x + 5)/(x - 3) = 4$. H [4]

E · SHOW THAT

20 Show that $(x^2 + 4x)/(x^2 + 2x - 8)$ simplifies to $x/(x - 2)$. H [3]

21 Show that $(2x^2 + 6x)/(x^2 - 9)$ simplifies to $2x/(x - 3)$. H [3]

22 A student writes $(x + 6)/(x + 3) = 2$ by "cancelling". Explain the mistake. H [3]

Marks are a guide to how much working an exam would expect. Questions written by Bloom Education in the style of GCSE (AQA, Edexcel, OCR) and Cambridge IGCSE papers.

Check your work

1 $2 \frac{6(x+2)}{3(x+2)}$ over $3(x+2)$: cancel $(x+2)$, leaving $6/3 = 2$.

2 $x+3$ Factorise numerator as $(x-3)(x+3)$. Cancel $(x-3)$. Answer: $x+3$.

3 $(x-4)/(x-2)$ $(x-4)(x+4)$ over $(x+4)(x-2)$: cancel $(x+4)$.

4 $(x+2)/(x+1)$ Factorise numerator as $(x+2)(x+3)$. Factorise denominator as $(x+1)(x+3)$. Cancel $(x+3)$. Answer: $(x+2)/(x+1)$.

5 $(x+3)/(x-1)$ Factorise numerator as $(x-5)(x+3)$. Factorise denominator as $(x-5)(x-1)$. Cancel $(x-5)$. Answer: $(x+3)/(x-1)$.

6 $3/2$ $18x$ over $12x$; cancel x , then $18/12 = 3/2$.

7 $4x^2$ Flip and multiply: $6x/5 \times 10x/3 = 60x^2/15 = 4x^2$.

8 $2/x$ $4x$ over $2x^2$; cancel $2x$, leaving $2/x$.

9 12 Flip and multiply: $8x/3 \times 9/(2x) = 72x/(6x) = 12$.

10 $(x+2)/x$ $x^2 - 4 = (x-2)(x+2)$, $x^2 + 3x = x(x+3)$; cancel $(x+3)$ and $(x-2)$.

11 $7x/12$ Find common denominator 12. Rewrite as $4x/12 + 3x/12$. Add numerators: $7x/12$.

12 $(7x-2)/12$ Find common denominator 12. Rewrite as $4(x+1)/12 + 3(x-2)/12$. Expand and add: $(4x+4+3x-6)/12 = (7x-2)/12$.

13 $(10-x)/12$ Find common denominator 12. Rewrite as $3(x+2)/12 - 4(x-1)/12$. Expand and subtract: $(3x+6-4x+4)/12 = (-x+10)/12$.

14 $(5x-1)/((x+1)(x-2))$ $(2(x-2)+3(x+1))$ over $(x+1)(x-2) = (5x-1)/((x+1)(x-2))$.

15 $(x+8)/(x(x+2))$ $(4(x+2)-3x)$ over $x(x+2) = (x+8)/(x(x+2))$.

16 $x=2$ or $x=-0.5$ $2(x+1)+3x=2x(x+1)$, so $2x^2-3x-2=0=(2x+1)(x-2)$.

17 5 Multiply both sides by $(x-1)$: $x+3=2(x-1)$. Expand right: $x+3=2x-2$. Subtract x : $3=x-2$. Add 2: $5=x$.

18 7 Multiply both sides by $(x-4)$: $x+2=3(x-4)$. Expand right: $x+2=3x-12$. Subtract x : $2=2x-12$. Add 12: $14=2x$. Divide by 2: $7=x$.

19 $17/3$ Multiply both sides by $(x-3)$: $x+5=4(x-3)$. Expand right: $x+5=4x-12$. Subtract x : $5=3x-12$. Add 12: $17=3x$. Divide by 3: $17/3=x$.

20 $x/(x-2)$ $x(x+4)$ over $(x+4)(x-2)$; cancel $(x+4)$.

21 $2x/(x-3)$ $2x(x+3)$ over $(x-3)(x+3)$; cancel $(x+3)$.

22 $x+6$ and $x+3$ have no common factor, so nothing cancels Only whole factors cancel. Test $x=1$: $(1+6)/(1+3)=7/4$, not 2.

Exam checklist

- Factorise before cancelling to find common slices.
- Cancel whole brackets, not individual terms, like whole pizzas.
- Flip the second fraction when dividing to change the operation.
- Use a common denominator to add or subtract slices of the same size.
- Always put brackets around the second top to keep the operation clear.
- Check denominators aren't zero, as you can't divide by nothing.

Got stuck on a question?

- Find the section it belongs to and go through its worked example again, line by line.
- Put your answer back into the question. If it doesn't work, look for a sign or a step you skipped.
- Still stuck? Message us on Instagram (@bloom_education_international). We read every message.

Mocks coming up? **Your first lesson is free.**

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